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KATEDRA FINANCÍ

Aplikace metodologie reálných opcí u developerských projektů

Real option aplication in development projects

Student: Bc. Lenka Musialová

Vedoucí diplomové (bakalářské) práce: prof. Dr. Ing. Zdeněk Zmeškal

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Místopřísežně prohlašuji, že jsem celou práci včetně příloh vypracovala samostatně.

V Ostravě 30. 4. 2010 Lenka Musialová

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Introduction

Real estate development is business, with activities that range from the renovation and re-lease an existing buildings to the purchase of raw land and the sale of parcels. Developers are the coordinators of the activities. Developers buy land, finance real estate deals, build or have builders to build projects, create, imagine, control and orchestrate the process of development from the beginning to the end. Developers usually take the greatest risk in the creation or renovation of real estate and receive the greatest rewards. Developing is the key world, because we don´t built ourselves.

In real estate land value is propably the most fundamental topic. Land is the fundamental characteristic of real estate. The nature of land valuation helps to define the investment characteristics of most real estate assets. The hold of the land has been recognized as an option to develop a completed building at a future date.

The traditional capital budgeting for a land development investment oportunity involves determining the project´s net present value. The first step in this analysis is to forecast cash flows over time and discount it at the appropriate required rate-of-return. The NPV is the diference between the present value of the expected future cash flows and the present value of the expected cost outflows. The decission rule is to accept the project when the NPV is zero or positive.

This simple NPV decision rule ignores the changing dynamics of the actual marketplace. For example future cash flows may differ from what developers originally expect, and developers may have flexibility to change its original strategy. This flexibility to adapt its future actions to market conditions is termed “real option“. Real-options analysis takes into account that investment flexibility.

An option is defined as a right without obligation to obtain something of value upon the payment or giving up of something else of value. The most important is that the real option, that is viewed as giving land its value, is the land development option. Our objective is to evaluate the value of land for a development project. We can say the right without obligation to develop the land upon payment of construction costs.

The main aim of the diploma thesis is to evaluate the value of land for the development project using the real option theory.

The first chapter is simple introduction and the adumbration of the diploma thesis.

The second chapter is focused on general characterictic of financial and real options.

The third chapter includes fundamentals about real estate system, development industry and is focused to land value, the fundamental characteristic of real estate. It is described how to evaluate the land using the real option theory, call option model.

The fourth chapter is to evaluate the land using the option valuation theory.

The fifth chapter involves the conclusion and comparison with more traditional discounted cash flow approach.

2. Types and description of option methods pricing

The real options can be explain using the theory of financial options. Financial options have much in common, but these two approaches, however, there are some differences. The main differences are shown in Tab. (2.1).

FINANCIAL OPTION	REAL OPTION
Value of underlying asset	Present value of subsequent cash flow
Exercise price	Investment cost
Time expiration of option	Life of project
Volatility of underlying asset	Volatility of project cash flow
Risk-free rate	Risk-free rate

Table 2.1 Comparison of financial and real options parameters

2.1 Financial options

In finance, an option is a contract between a buyer and a seller that gives the buyer of the option the right, but not the obligation, to buy or to sell a specified asset (*underlying asset*) on or before the option's expiration time, at an agreed price, the *strike price*. For granting the option, the seller collects a payment (the *premium*) from the buyer. It is refered to as "selling" or "writing" the option.

Underlying asset can be financial asset (stock price, stock index, the price of bonds, exchange rate, etc.) or non-financial factors, we are talking about real options (cash flow or other risk factors). It is an asset or financial instrument upon which a derivative depends. The price of the underlying determines the price of the derivative that is linked to it. As the underlying's price fluctuates so will the value of the derivative.The letter *S* indicates the underlying asset in option terminology.

Strike (exercise) price is the price at which an asset will be bought (call) or sold (put) under an option contract. The exercise price is determined at the time the option contract is formed. It can be marked with the letter *X*.

Option price (option premium) called the letter *C*, is a market price option contract that ensures the right of option. It can be expressed as the purchase price of an option contract and the possitive difference between the pirce of a security and its face vaue or par amount.

Expiration date is the date on which the right of option expires and ends its validity.
Derivative is a security whose value is derived from the values of other assets. Derivatives are therefore generally reffered as derivative securities.
The pay-off function, also known as *intristic value* is amount of the payment at the time of use for the European-type option to the point of implementation.
Possible positions in the option contract:

- a call option buyer,
- selling a call option,
- buyer of a put option,
- seller of a put option.

Option holder is the party who pays a premium for the right to buy or sell the underlying asset under an option contract up to maturity and has not yet exercised or sold that right.
Option writer is the seller of either a call or put option who receives a payment (premium) from the buyer obligating the seller to fulfill the contract if and when the buyer (holderÚ exercises the option.

If is the price of the underlying asset such as shares, upon maturity options higher than exercise price, the call option has positive value and is located in the zone in-the-money. Put option in this situation have not value and is out-of-money. This can be expressed by:

$$S - X > 0. \tag{2.1}$$

Another situation occurs when due to the expiration date of the option is the price of the underlying equal to the strike price. The value of call and put option is zero and we say that it is at-the-money.

$$S - X = 0 \tag{2.2}$$

If the exercise price, in the date of expiration, exceeds price of underlying, call option has not value and we can say that the situation is out-of-the-money. On the contrary, the value of a put option is positive and is in-the-money. For this situation the following situation holds,

$$S - X < 0, \tag{2.3}$$

where S underlying asset, X strike price.

2.1.1 Long call

A trader who believes that a stock's price will increase might buy the right to purchase the stock rather than just buy the stock. He would have no obligation to buy the stock, only the right to do so until the expiration date. If the stock price at expiration is above the exercise price by more than the premium (price) paid, he will profit. If the stock price at expiration is lower than the exercise price, he will let the call contract expire worthless, and only lose the amount of the premium. A trader might buy the option instead of shares, because for the same amount of money, he can control a much larger number of shares.

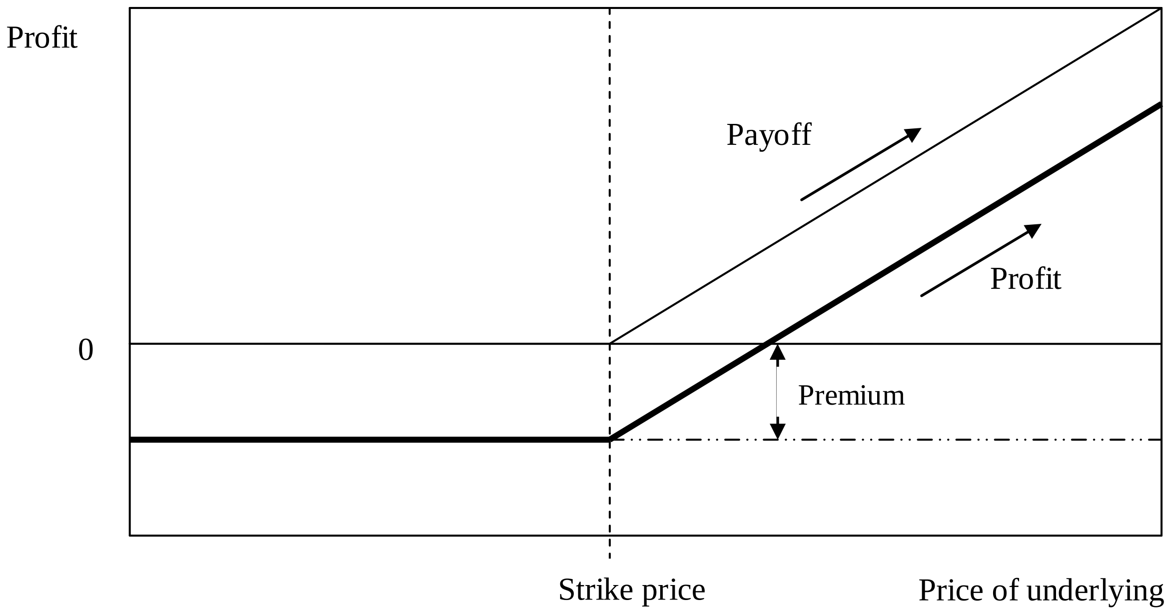
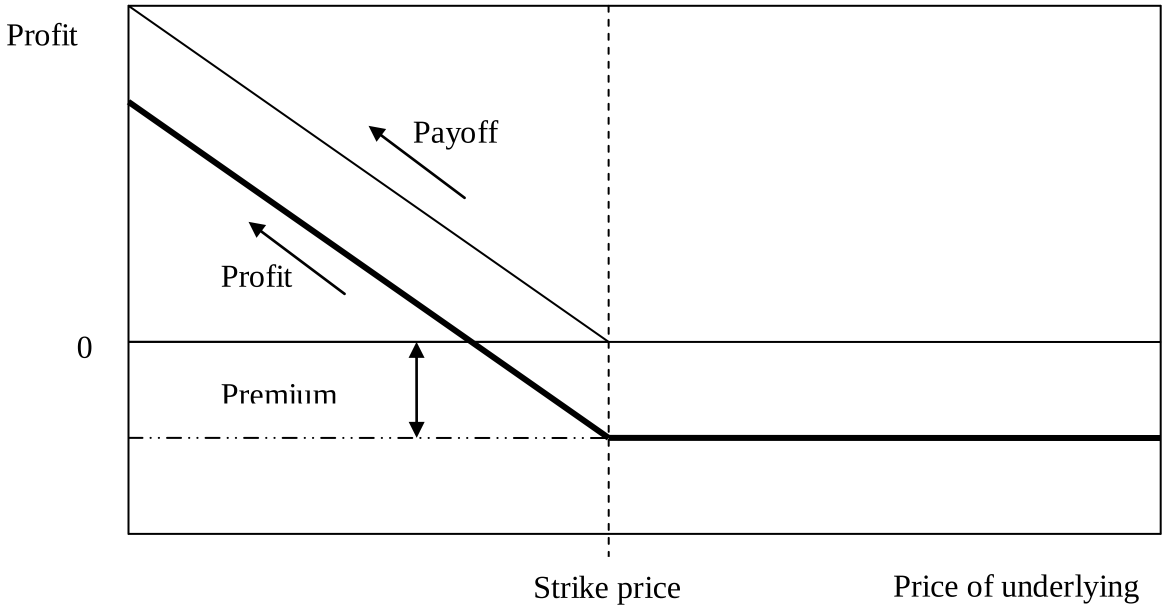


Figure 2.1 Payoff diagram (long call)

2.1.2 Long put

A trader who believes that a stock's price will decrease can buy the right to sell the stock at a fixed price. He will be under no obligation to sell the stock, but has the right to do so until the expiration date. If the stock price at expiration is below the exercise price by more than the premium paid, he will profit. If the stock price at expiration is above the exercise price, he will let the put contract expire worthless and only lose the premium paid.



Figuret 2.2 Payoff diagram (long put)

2.1.3 Short call

A trader who believes that a stock price will decrease, can sell the stock short or instead sell, or "write," a call. The trader selling a call has an obligation to sell the stock to the call buyer at the buyer's option. If the stock price decreases, the short call position will make a profit in the amount of the premium. If the stock price increases over the exercise price by more than the amount of the premium, the short will lose money, with the potential loss unlimited.

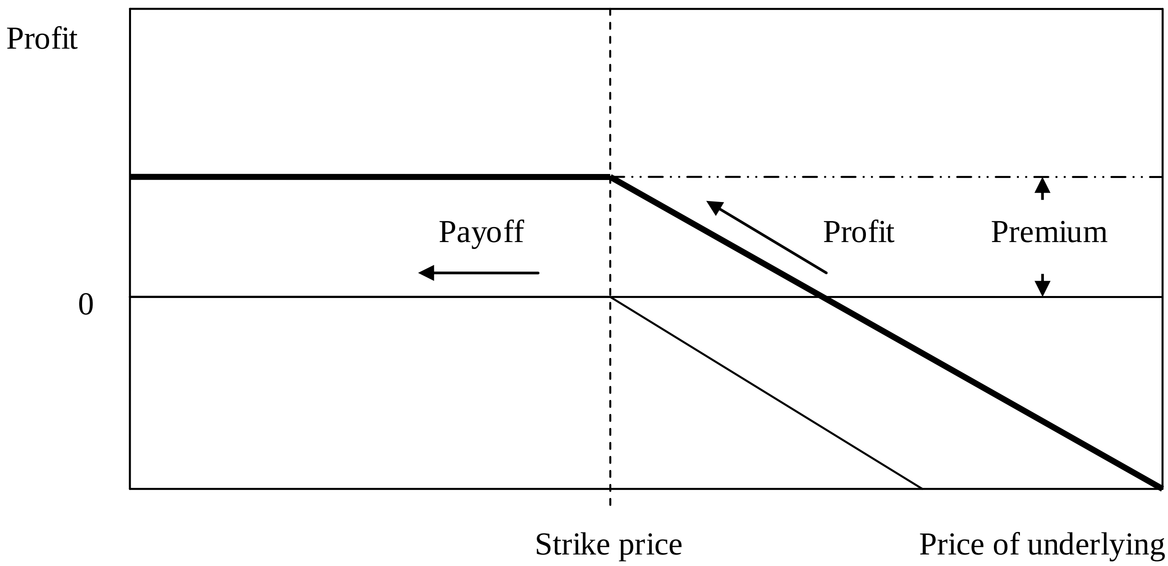


Figure 2.3 Payoff diagram (short call)

2.1.4 Short put

A trader who believes that a stock price will increase can buy the stock or instead sell a put. The trader selling a put has an obligation to buy the stock from the put buyer at the put buyer's option. If the stock price at expiration is above the exercise price, the short put position will make a profit in the amount of the premium. If the stock price at expiration is below the exercise price by more than the amount of the premium, the trader will lose money, with the potential loss being up to the full value of the stock.

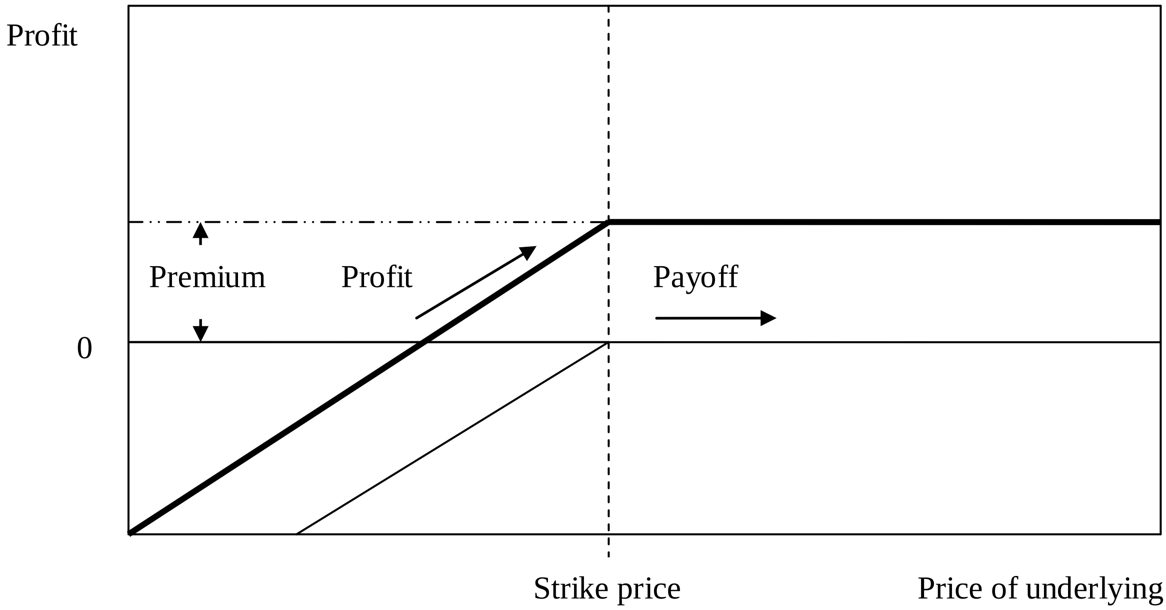


Figure 2.4 Payoff diagram (short put)

The table bellow sumarizes variables and their predicted effects on call and put prices.

	Effect on	
Factor	Call value	Put value
Increase in underlying asset´s value	Increases	Decrease
Increase in strike price	Decreases	Increases
Increase of variance in underlying asset	Increases	Increases
Increase in time to expiration	Increases	Increases
Increase in interested rates	Increases	Decreases
Increase in dividends paid	Decreases	Increases

Table 2.2 Summary of variables affecting call and put prices

2.2 Real options

The methodology of real options and its application in financial management firms in the current uncertain and high risk project management of paramount importance, due to Hull (2001). In its application the practical investment decision to work with flexibility, enabling changes and additional interventions already initiated projects in the event that market conditions. Managers managing the process of adaption to changed conditions are using this methodology to minimalize possible negative impacts. Since such intervation may involve a possitive change in terms of efficiency project implemented, it is necessary to include the options choice decission to the total value of the project, before begining of its implementation. Therefore the conclusions and recommendations of traditional methods and real options are often so different. The most important factors determining the price of real options are,

- *Underlying asset* – in the case of real optionn it is cash flow of project at time t, or, gross project value V_t. The higher value of the underlying asset, the higher value of call option, in the case of put option, the opposite is true.
- *Exercise price* – it is equal to the investment cost, which would have to be spent if call option were exercised, in the case of put options it meaans saved investment cost or, selling asset price.
- *Time to expiration* – during this time period option can be exercised. Usually exercising can appear whenever during the life of the project (American option), if the opportunity can be exercised only at pre-specified time, then it is Evropean option.
- *Volatility of underlying asset* – value of an option and the project is the higher, the higher is the risk of the underlying asset expressed in its price volatility. This feature refers both to call and put options.
- *Risk free rate* – the higher rate, the higher option value.

The basic types of real options for investment project, due to Dluhosova (2004),

- option to defer a project,
- option to expand a project,
- option to contract a project,
- option to abandon a project,
- option to temporarily shut down and restart the project.

2.2.1 Option to defer (delay) a project

This type of option is European call option. It enables managers temporarily to defer starting the project and profit from future information, which are over time resolved and were unknown at the outset of the decision. Managers defer the project with investment cost *I*, if project´s NPV is higher compared with its immediate starting. Function of intristic value of the option, *IV*, can be formally written as follows,

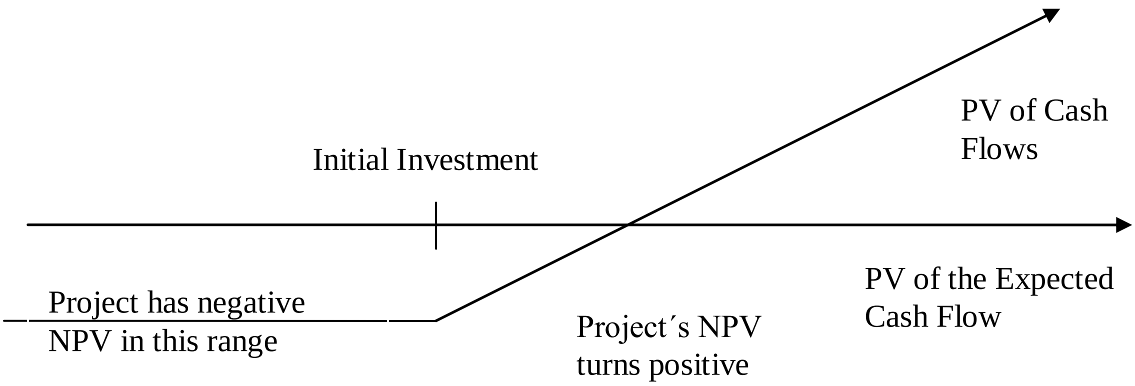
$$IV = Max\left[V_0 - I_0; \frac{1}{(1+r)^t}(V_t - I_t)\right], \tag{2.4}$$

where V_t is the gross project value (present value of subsequent cash flow discounted back to the time t).

Decision function is written this way,

$$F = \begin{Bmatrix} 1\, for(V_0 - I_0) < \frac{1}{(1+r)^t}(V_t - I_t); \\ 0\, for(V_0 - I_0) > \frac{1}{(1+r)^t}(V_t - I_t) \end{Bmatrix}, \tag{2.5}$$

where 1 means to defer a project, 0 means to start a project immediately.
This relationship can be presented in a payoff diagram of cash flows on the project, as shown in Figure 2.5.



Figuret 2.5 The option to defer a project

2.2.2 Option to expand a project

If the project has already been once undertaken, management have the possibility to make additional investment and expand the initial production if it turns out. In the option pricing terminology, firm has a call option on additional cash flow from extended part of project with exercise price equals to investment cost.

This type of option can be called as an European or whenever during the life of project as an American option.

Function of intristic value of the option, *IV*, can be formally written as follows,

$$IV_t = Max\left[V_t ; x \cdot V_t - I_{E,i}\right], \tag{2.6}$$

where $I_{E,i}$ is the investment cost for project expanding at time t and x is the scale expanding of the basic project.

Decision function is written in this way,

$$F = \left\{ \begin{array}{l} ExpandIfV_t < x \cdot V_t - I_{E,i}; \\ Ma\,\mathrm{int}\,a\,\mathrm{int}\,heInitialScaleIfV_t > x \cdot V_t - I_{E,i} \end{array} \right\}. \tag{2.7}$$

This relationship can be presented in a payoff diagram of cash flows on the project, as shown in Figure 2.6.

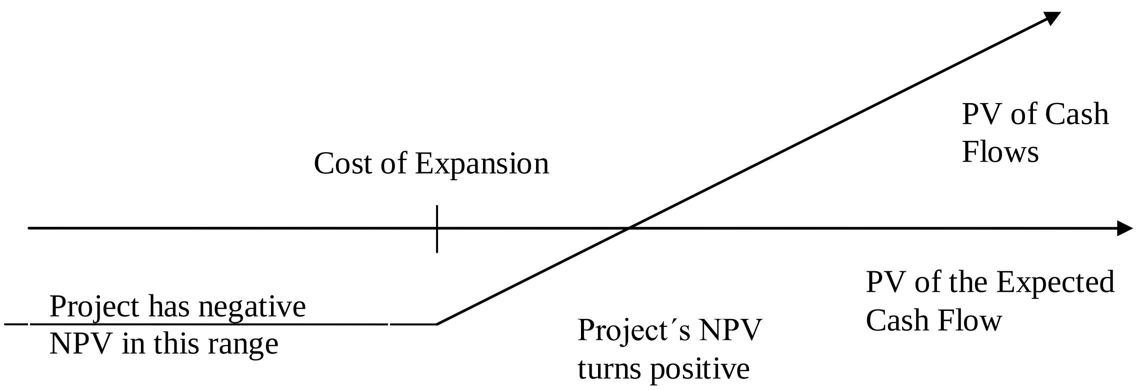


Figure 2.6 The option to expand a project

2.2.3 Option to contract a project

In this case, management has the option to contract the initial scale of the production and sale a part of the project, if the conditions turned out to be less favourable than those expected at the beginning of the investment process. Thus can be seen as a put option on the part of the initial project and cash flow generated by this part, which can be contracted with exercise price equals to the saved investment cost.

This type of option can be defined both as a Evropean and as an American option.

Function of intristic value of the option, *IV*, can be formally written as follows,

$$IV_t = Max\left[V_t ; I_{C,t} - y \cdot V_t\right], \tag{2.8}$$

where $I_{C,t}$ is the investment cost which can be saved if the project is contracted at time t and y is the proportional part of the cash flow from contracted initial project V_t.

Decision function is written this way,

$$F = \begin{Bmatrix} Contract If V_t < I_{C,t} - y \cdot V_t; \\ Ma \text{int} a \text{int} he Initial S cale If V_t > I_{C,t} - y \cdot V_t \end{Bmatrix}. \quad (2.9)$$

2.2.4 Option to abandon a project

If the conditions turned out to be unfavourable, management may have option to abandon the ptoject in exchavge for tis salvage price A_t before tis expected life. In other words, management has a put option on the gross value of the project with exercise price equals to the salvage or resale value, which can be written as,

$$IV_t = Max[V_t; A_t - V_t]. \quad (2.10)$$

Management can continue the operation or abandon the project if, at time *t*, the salvage or resale value is higher than the subsequent cash flow from continuing the project discounted back to the time *t*.

Decision function is written this way,

$$F = \begin{Bmatrix} Abandon If V_t < A_t; \\ Ma \text{int} ain The Operation If V_t > A_t \end{Bmatrix}. \quad (2.11)$$

This relationship can be presented in a payoff diagram of cash flows on the project, as shown in Figure 2.7.

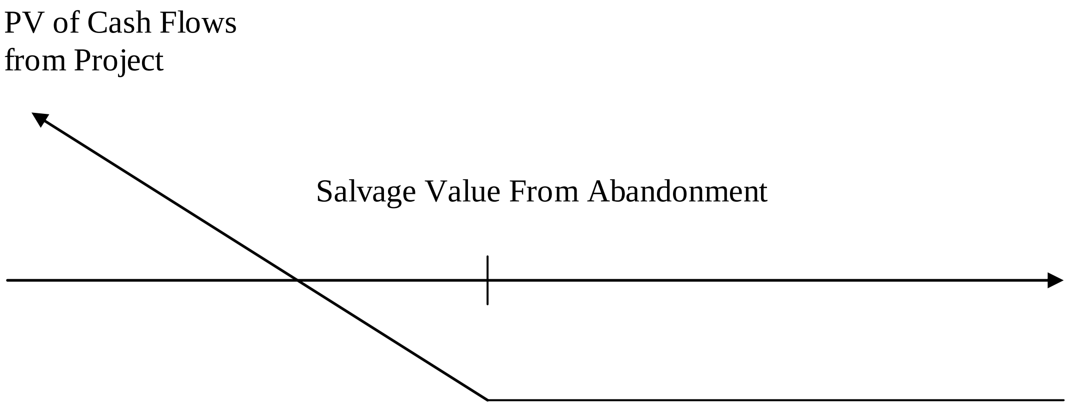


Figure 2.7 The option to abandon a project

2.2.5 Option to temporarily shut down and restart the project

In the case the revenue *R* in a given year is not sufficient to cover variable cost of the production, management may have the option to temporarily shut down the production or simply not to operate. Operation in a given year may be viewed as a call option on the production by paying variable cost *VC* as the exercise price.

Function of intristic value of the option, *IV*, can be formally written as follows,

$$IV_t = Max(R_t - VC_t;0) - FC. \tag{2.12}$$

Decision function is written this way,

$$F = \begin{Bmatrix} TemporarilyShutDownIfR_t < VC_t; \\ ContinueTheOperationIfR_t > VC_t \end{Bmatrix}.$$

2.3 Real option methodology

Most traditional methods used to decide whether or not an investment project should be undertaken are based on discounted cash flow (DCF) methods, builds on a relationchip between present value and future value. The Net Present Value (NPV), particular version of DCF, is the most well known method used in decision analysis. The NPV rule is simpple: invest at once, if the NPV is positive, otherwise reject the project.

$$NPV = \sum_{n=0}^{N} \frac{CF_n}{(1+i)^n} - CE\,, \tag{2.13}$$

where NPV is the net present value of the project, CF_n cash flow at each year, i requiered rate of return, n the lifetime of the project, CE capital expenditure at the beginning of investment.

In an advanced form of the previous relationship can be broken down as,

$$NPV = \frac{CF_1}{(1+i)} + \frac{CF_2}{(1+i)^2} + ... + \frac{CF_n}{(1+i)^n} - CE\,. \tag{2.14}$$

The interpretation of results can be as follows,

$NPV > 0$ means that the discounted cash flows exceeds the capital expenditure. The project is acceptable to an investor.

$NPV < 0$ means that the discounted cash incomes are lower than capital expenditure. In this case the project is not acceptable.

$NPV = 0$ means that in this situation the investor is indifferent to investing.

But this method has several limitations. The NPV rule is based on basis invest now or never. But it is not reality. Managers have to manage investments by changing subsequent plans and past decisions in response to changing market conditions. Some decisions have to be done after some uncertainties are resolved.

Traditional DCF approach assumes a single decision with fixed outcomes and all decisions are made at the beginning of the investment process without the ability to change it or make new ones. The real option approach considers multiple decision pathways as a consequence of high uncertainty with management flexibility in choosing the optimal strategies or options along the way when new informations becames available. Real option theory assumes dynamic series of decisions, where management has the flexibility to revise and adapt past decisions in response to actual real conditions.

This flexibility can be modeled as an call and put options we spoke above. These options are valuable and create great part of total project value. Real option theory enables these options defined, quantify them and incorporate them into decision-making process. The value of option is called option premium, added to the value of project can lead to the situation, where these projects can be started, which with using of passive methods would be normally rejected. The basic advantage of real option methodology is that there is no need to complicatelly calculate risk-adjusted cost of capital but only risk free rate is necessary to know and use.

Option theory have three differences versus the DCF theory,

- *flexibility* is the ability (option) to defer, abandon, expand, contract a project. Because the NPV rule is defined as passive, values these options at zero, while the real option approach would correctly allocate some project value into these future options,
- *contingency* means that investments are contingent on the success of today´s investment, managers can make investments today (even those with negative NPV) to access future possibilities,
- *volatility*. Investments with greater uncertainty have higher option value. Normally higher volatility means higher risk, higher discount rate and lower present value. Inoption theory, higher volatility (because of asymetric payoff schemes) leads to higher option vaue.

There are a variety of methodologies and approaches to determining the value of the option, the most common methodologies,

- binomial option pricing model,
- Black Scholes option pricing model,
- Monte carlo simulation.

2.3.1 Binomial option pricing model

An options valuation method developed by Cox, et al, in 1979. The binomial option pricing model uses an iterative procedure, allowing for the specification of nodes, or points in time, during the time span between the valuation date and the option's expiration date.

That method calculate possible paths that might be followed by the underlying asset's price over the life of the option. The model works by dividing the time to expiration into a number of time intervals and over each time interval, the model assumes that the price of the underlying moves up or down to certain values. Then from the up or down prices at the next time step, the up and down price is calculated for each scenario until the expiry date. The magnitude of these moves is determined by the volatility of the underlying and the length of the time interval.

Prerequisites of binomial option pricing model are as follows,

- the absence of arbitrage opportunities (ie, inability to achieve risk-free profit),
- the force of law of one price (if you have two identical assets in the future pay the same, function, then assuming the impossibility of arbitrage are now the same price)
- the existence of a perfect market (no transaction costs, taxes, restrictions on short sales),
- infinite divisibility of the underlying assets,
- gender proceeds of any asset risk-free rate,
- discrete time intervals,
- risk-neutral propability.

Binomial model is stochastic (discontinuous) option pricing model, which is presumed that the price of the underlying asset develops discreet manner. This is a binomial model, which means that from one starting point, S_0, may occur only two situations, growth and decline of the underlying asset. Index of up movement, u, and down movement, d, are derived from the underlying asset price volatility and can be determined as follows,

$$u = e^{\sigma \cdot \sqrt{dt}}, \tag{2.15}$$

$$d = e^{-\sigma \cdot \sqrt{dt}}, \tag{2.16}$$

where *u* up movement, *d* down movement, *σ* standard deviation, *dt* length of time interval.
The product of growth and decline in the index must be zero.
If it is a multiplicative (geometric) process, the value of the underlying asset evolves according to Figure 2.8.

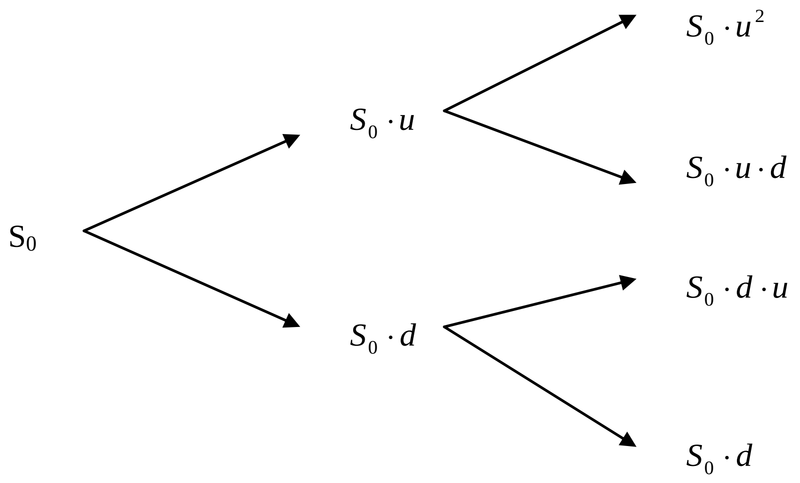


Figure 2.8 General formulation for binominal price path

Binomial option pricing model is used for both options of European type and American type of option. There are two approaches for options pricing, replication and hedging strategies.

Replication strategy is based on the assumption that we can build a portfolio of underlying asset and risk-free asset, that the value of the portfolio will replicate the value of the option. Replication portfolio value at time t is as follows,

$$a \cdot S_t + B_t = C_t, \tag{2.17}$$

where a is the quantity of the underlying asset, S_t is the value of the underlying asset, B_t amount of risk-free asset and C_t the option price at time t.

Portfolio value at time t + dt, the time 1, for the up movements and down movements in prices of the underlying asset is as follows,

up movement $$C_1^u = a \cdot S_1^u + B \cdot (1 + r), \tag{2.18}$$

down movement $$C_1^d = a \cdot S_1^d + B \cdot (1 + r). \tag{2.19}$$

At the time of the option expiration the price equals intrinsic value,

$$C_1^u = IV_1^u \text{ or } C_1^d = IV_1^d, \tag{2.20}$$

System of three equations (2.18) (2.19) and (2.20) for the three unknowns to obtain a general formula for calculating an option,

$$C_0 = (1+r)^{-1} \cdot \left\{ C_1^u \left[\frac{(1+r) \cdot S_0 - S_1^d}{S_1^u - S_1^d} \right] + C_1^d \left[\frac{S_1^u - (1+r) \cdot S_0}{S_1^u - S_1^d} \right] \right\}, \tag{2.21}$$

$$C_0 = (1+r)^{-1} \cdot \left[C_1^u \cdot p + C_1^d \cdot (1-p) \right], \tag{2.22}$$

$$C_0 = (1+r)^{-1} \cdot \left[E(C_1) \right], \tag{2.23}$$

$$C_0 = PV\left[E(C_1) \right], \tag{2.24}$$

where $(1+r)^{-1}$ is discount rate, p risk-neutral propability.

From this relationship implies that the price of European option is determined as the present value of the median price of an option in the next period based on risk-neutral probability. To determine the price of American option is the following formula,

$$C_0 = \max\left[IV_0 ; (1+r)^{-1} \cdot \left(C_1^u \cdot p + C_1^d \cdot (1-p) \right) \right], \tag{2.25}$$

due to condition of no arbitrage,

$$d \langle (1+r)^{dt} \langle u . \tag{2.26}$$

Price based on option hedging strategy is based on the assumption that you can create a portfolio consisting of h-shares of the underlying asset and short positions in call options so that the yield of the portfolio in any development value of the underlying asset is risk free.

The value of the hedged portfolio at time t,

$$\Pi_t = h \cdot S_t - C_t , \tag{2.27}$$

where Π_t is the value of portfolio a h the amount of underlying asset.

The value of the portfolio at the time t+dt,

$$\Pi_{t+dt}^u = h \cdot S_{t+dt}^u - C_{t+dt}^u , \tag{2.28}$$

$$\Pi_{t+dt}^d = h \cdot S_{t+dt}^d - C_{t+dt}^d . \tag{2.29}$$

Ensure against changes in the price movement of the underlying asset means that at the end of the period, the portfolio will be the same whether the price change in any direction. This situation is as follows,

$$h \cdot S_{t+dt}^u - C_{t+dt}^u = h \cdot S_{t+dt}^d - C_{t+dt}^d . \tag{2.30}$$

Hedge ratio (*h*) is equal,

$$h = \frac{C^u_{t+dt} - C^d_{t+dt}}{S^u_{t+dt} - S^d_{t+dt}} = \frac{\Delta C}{\Delta S}. \tag{2.31}$$

Portfolio yield should be risk-free, thus holds,

$$h \cdot S_t - C_t \cdot (1+r)^{dt} = h \cdot S^u_{t+dt} - C^d_{t+dt}, \tag{2.32}$$

$$h \cdot S_t - C_t \cdot (1+r)^{dt} = h \cdot S^d_{t+dt} - C^d_{t+dt}. \tag{2.33}$$

The price of European option is as follows,

$$C_t = h \cdot S_t - \left(h \cdot S^u_{t+dt} - C^u_{t+dt}\right) \cdot (1+r)^{-dt}, \tag{2.34}$$

$$C_t = h \cdot S_t - \left(h \cdot S^d_{t+dt} - C^d_{t+dt}\right) \cdot (1+r)^{-dt}. \tag{2.35}$$

It is the main trend in the option pricing method. The advantage is in their simplicity of implementation and interpretation. Using a binomical model we obtain a high degree of flexibility in the modeling of new decisions ans applications of various types of options. The results obtained using the binomial model is generally closer to the results calculated used an equation approaches. It is recomended to applicate both approaches simultaneously, which in turn allows their comparison.

It was an introdution of binomial option pricing model based on risk-neutral propability, but alse we can use the real propability.

<u>Real propability</u>

Opposite the use of risk-neutral propability, we can use the “real propability“, due to Arnold, T., & Crack, T. (2003). It is more applicable to price options in real world. For example, the propability of success of a real-option project, the probability of default on a corporate bond, the probability that an American-style option will fnish in the money. Similarly, if higher moments (skewness and kurtosis) play a part in the asset pricing model, then practical problems arise because the variance and higher moments can differ between the real and risk-neutral worlds.

The option price follows three different ways, binomial model, CAPM and certainty-equivalence, the way how to compute the option price is described using the equations bellow,

$$C_0 = \frac{1}{r_f}\left[E(C_T) - \left(\frac{C_u - C_d}{u-d}\right)\left(E(r_s) - r_f\right)\right], \tag{2.35}$$

$$u = e^{\sigma \cdot \sqrt{T}}\,, \tag{2.36}$$

$$d = e^{-\sigma \cdot \sqrt{T}}\,, \tag{2.37}$$

where r_s is the discretely compounded return on the underlying asset, r_f risk free rate.

$$C_0 = e^{-rfT}\left[E(C_T) - \left(\frac{C_u - C_d}{e^{\sigma\sqrt{T}} - e^{-\sigma\sqrt{T}}}\right)\cdot\left(e^{ksT} - e^{r_f T}\right)\right], \tag{2.38}$$

$$r_s = \frac{S_T}{S_0}\,, \tag{2.39}$$

where $e^{r_f T}$ annualized continously compouded risk free rate, e^{ksT} annualized continously compouded expected return on the underlying asset.

Note that although equation (2.35) involves discounting at the risk-free rate, this is not risk-neutral pricing. There is no change of probability measure. The expected cash flow $E(C_T)$ is in the real world, not a risk-neutral world, and it is not directly discounted at the risk-free rate. Rather, the risk-adjusted expected cash flow (the certainly equivalent) is discounted at the risk-free rate. This risk-adjusted expected cash flow is the real-world expected cash flow less a risk premium.

The real propability is computed as follows,

$$p = \left(\frac{e^k S^T - d}{u - d}\right), \tag{2.40}$$

where e^k is the continously-componded annualized risk-adjusted expected rate of return for the underlying asset, S is the current value of underlying asset, T is the proportion of a year for one stage of a binomial tree.

The value of the option is calculated as follows,

$$C_{i,j} = e^{-r_f T}\left[\left(pC_{i+1,j} + (1-p)C_{i,j+1}\right) - \left(\frac{C_{i+1,j} - C_{i,j+1}}{e^{\sigma\sqrt{T}} - e^{-\sigma\sqrt{T}}}\right)\left(e^{k_s T} - e^{r_f T}\right)\right], \tag{2.41}$$

where i is the number of upward price movement, j is the number of downward price movement.

2.3.2 Black Scholes model

The Black Scholes Model is one of the most important concepts in modern financial theory. It was developed in 1973 by Fisher Black, Robert Merton and Myron Scholes and is still widely used today, and regarded as one of the best ways of determining fair prices of options.

Prerequisites of binomial option pricing model are as follows,

- continuous time,
- assumption of ideal capital market,
- price of the underlying asset evolves according to geometric Brownian motion, price changes are treated using a log-normal distribution,
- prices are independent of expected returns,
- a constant risk-free rate
- constant volatility,
- the payment of dividends is ignored,
- valuation of European options.

The price of European call option is given as follows,

$$c = S_0 \cdot N(d_1) - e^{-r \cdot dt} \cdot X \cdot N(d_2), \tag{2.36}$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right) \cdot dt}{\sigma \cdot \sqrt{dt}}, \tag{2.37}$$

$$d_2 = d_1 - \sigma \cdot \sqrt{dt}, \tag{2.38}$$

where c is the price of Evropean option, S_0 price of underlying asset, X strike price, r risk-free rate, dt time to maturity, σ standard deviation of underlying asset, $N(d_1)$ and $N(d_2)$ determine the value of the distribution function of standard normal distribution and $e^{-r \cdot dt}$ is continuous discount factor.

The price of European call option as given as follows,

$$p = e^{-r \cdot dt} \cdot X \cdot N \cdot (-d_2) - S_0 \cdot N \cdot (-d_1). \tag{2.39}$$

Put-call parity is a relationship between the prices of European put and European call options, is given by the following formula,

$$c + e^{-r \cdot dt} \cdot X = p + S_0. \tag{2.40}$$

If both European call and put options have equal input data (the underlying asset, strike price, time to maturity, volatility and risk-free interest rate) the price knowledge of one of them to determine the price of the other.

2.3.3 Monte Carlo simulation

A problem solving technique used to approximate the probability of certain outcomes by using a large number of repeated calculations, called simulations (a formal trial and error), using random variables thus creating a large number of scenarios. Monte Carlo simulation uses historical interest rate volatilities to generate the large number of interest rate paths needed to simulate (imitate) interest rate sensitivity. Monte Carlo simulation is one of the best tools to measure interest rate risk in particular for option valuation.

3. Description of real option in real estate

3.1 Real estate system

There are three major components of real estate system: *the space market, the asset market* and *the development industry*. The major elements and linkages among these three components presents chart bellow.

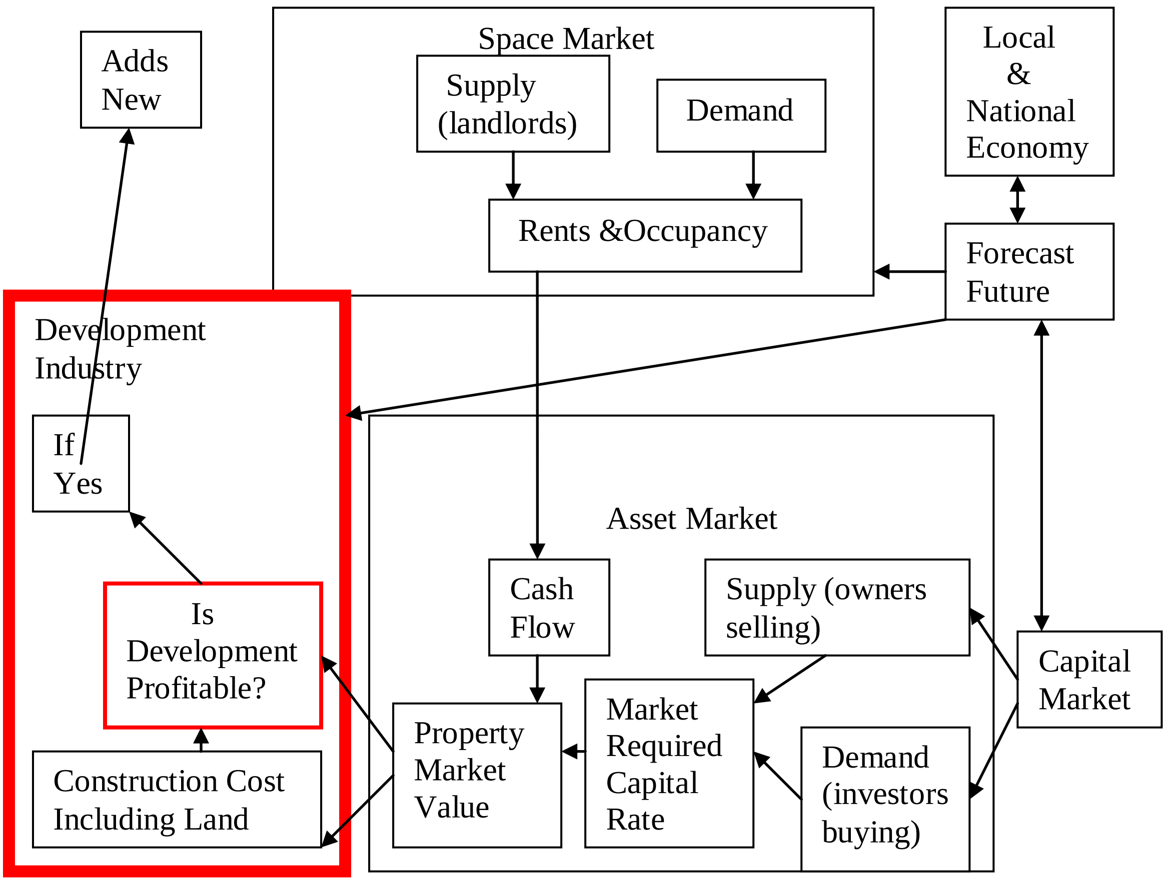


Figure 3.1 Real estate system – Interaction of the space market, ssset market, development IndustrY, due to Geltner (2007)

The space market is the market for the usage real property (land and built space). It is also referred to as the real estate market as the rental market. On the demand side are individuals, households, and firms that want to use space. On the supply side of the space market are real estate owners who rent space to tenants. The price of the right to use space for a period of time is called *rent*.

The real estate *asset market* is the market for the ownership of real estate assets. Real estate assets consist of real property, that is, land parcels and the buildings on them. Therefore, this market is often called property market.

The real estate *development industry* is the engine of interpreneurial activity that assembes and applies the financial and physical resources to construct new built space. Development is a complex of creative function, displays great vision considerable risk-taking on the part of the developer.

3.2 Four-quadrant model

As ilustrated in the Figure 3.2, the rent and the cap rate are determinants of the value of built property in the demand side of the real estate market. Then the value of the built property as a result of relevant rent and cap rate is a determinant of new construction in the supply side of the real estate market. In order to create possible realistic uncertain variable scenarios, designers need to know volatility of the variables. The data of volatility is usually acquired from observing historical performance data. In the case of real estate development, it is usually to obtain the data set of historical volatility of rent, cap rate and value of built property. Because the objective in this thesis is to examine the effect of real options analysis in the supply side of the real estate market, it is a reasonable shortcut to use the value of built property as a single source of uncertainty.

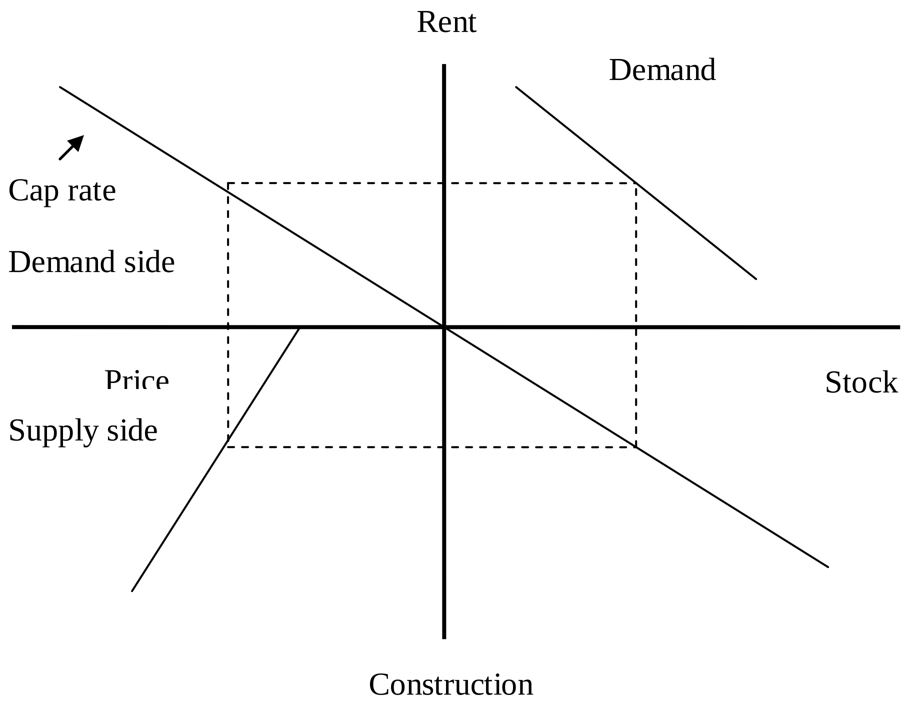


Figure 3.2 Four quadtant real estate market model, due to Wheaton (1996)

3.3 The development decision-making process

Development decision making in the private sector could typically be described by one of two situations*: a site looking for a use, or a use looking for a site*. In the former case the site is already under the control of the developer and the analysts undertakes what is, the highest use. It is not uncomon for developers or land speculators to buy and hold land when it is cheaper and not yet ready for development.

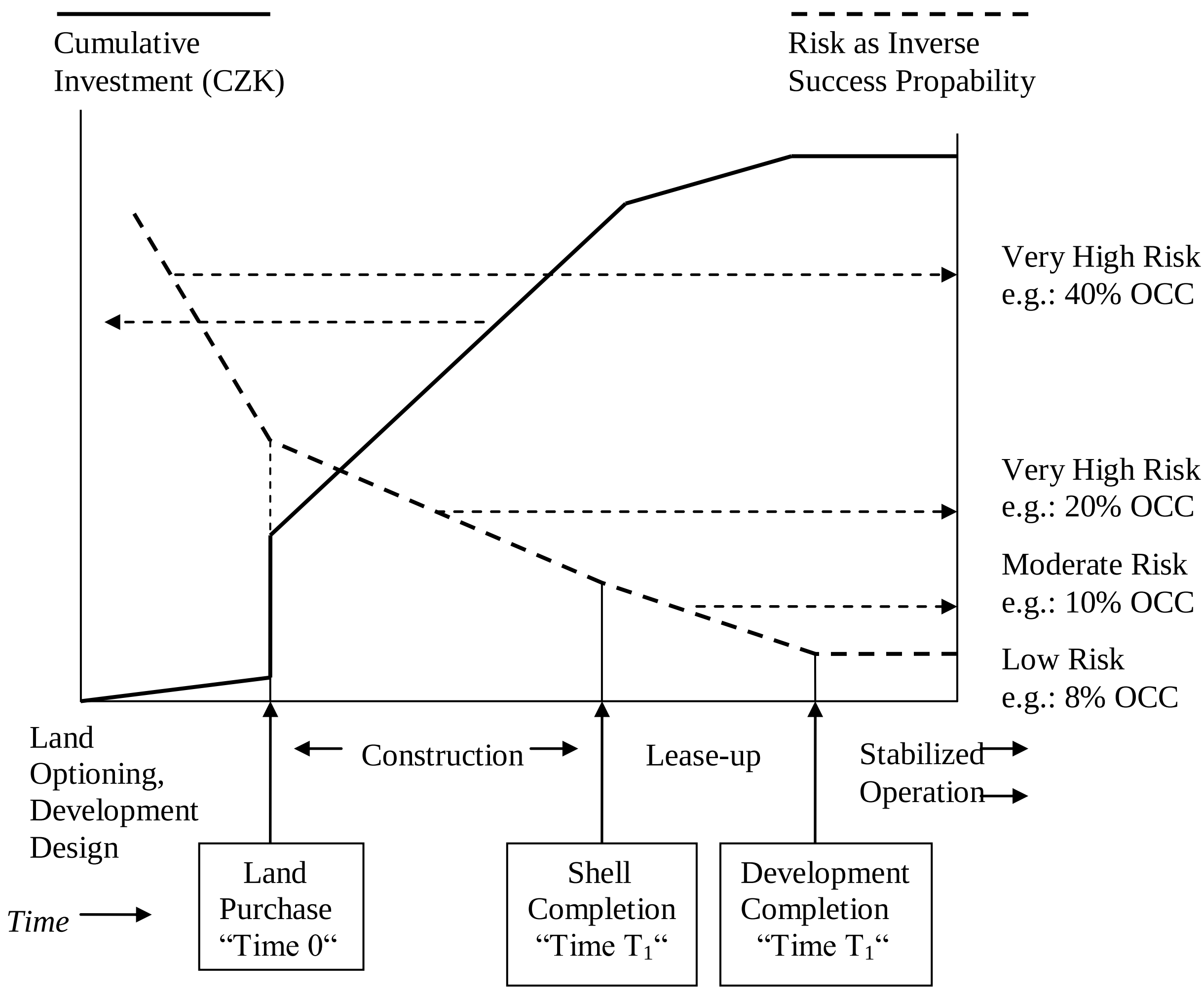


Figure 3.3 Development Project Phases, due to Geltner (2007)

The horizontal axis represents the time, can be devided into phases. The first phase can be viewed as a *preliminary* phase. This is the most creative and enterpreneurial time in the evolution of the project. This first phase involves optioning and assembly of separate land parcels, the obtaining of necessary permits, the design of the development project and include a highest and best use analysis of the project.

This preliminary phase may take anywhere from a few months to more than a decade and does not always succeed. The development project may ultimately not be approved or may not prove economically, financially, or administratively feasible. This is why this is the riskiest phase in a development process with high opportunity cost of capital. This may be viewed as land speculation investment rather that as real estate development project investment.

At the end of the preliminary phase the necessary land will be purchased or otherwise acquired, to give the developer the right to proceed with construction. Depends on when and how the land has actually been acquired, from the perspective of the evaluation of the development project as an investment, the econimic "opportunity cost" of the value of the land is incurred at the time of the start of construction. Thus at *Time 0* the land is commited to the construction project and we can say it is a land speculation with real options characteristics. The opportunity cost is the economic value of the land, what it could be sold for at *Time 0*, after it has been assembled and permited and a project design developed.

The development investment begins with the incurring of the land opportunity cost at the beginning of the construction phase, paying for the construction of the building(s) on the site. This phase is still quite risky, though less so than the preliminary phase. Development investments are risky for two reasons. If the project is speculative, then it is not known for certain what occupancy will be achieved within what period of time and at what rental rate. The second reason why development investments are risky as that they contain operational laverage. Even if there is no financial laverage and even if there is no lease-up risk, development still have operational laverage in the sence of having high fixed costs relative to potentionally variable revenues.

Next phase is described as lease–up. This is the phase after major construction has been completed, during which the space is leased and occupied by its users, this involves some finishing construction work that customizes the space in the building.

The construction and lease-up phases together represents the development project, the subject of investment analysis in this thesis. These development phases may take anywhere from a few months to several years. In larger projects, the development may ofer to be devided into separate stages, in which there remains flexibility about when to commence the later stages of the development. In such cases, the development project retains real options characteristics.

At the end of the development project, the result is an asset in stabilized operation. The project is completely or nearly leased up, and operating with profitability.

3.4 Real options and land value

Real estate development is business, with activities that range from the renovation and re-lease of existing buildings to the purchase of raw land and the sale of improved parcels to others. Developers are the coordinators of the activities, converting ideas on paper into real property. Developing is the key word, because we don´t built ourselves.

Developers buy land, finance real estate deals, build or have builders build projects, create, imagine, control and orchestrate the process of development from the beginning to end, Brueggeman (2008). Developers usually take the greatest risk in the creation or renovation of real estate and receive the greatest rewards. Typically, developers purchase a tract of land, determine the marketing of the property, develop the building program and design, obtain the necessary public approval and financing, build the structure, and lease, manage, and ultimately sell it. Developers work with many different counterparts along each step of this process, including architects, city planners, engineers, surveyors, inspectors, contractors, leasing agents and more. Purchasing unused land for a potential development is sometimes called speculative development. Subdivision of land is the principal mechanism by which communities are developed. Technically, subdivision describes the legal and physical steps a developer must take to convert raw land into developed land. Subdivision is a vital part of a community's growth, determining its appearance, the mix of its land uses, and its infrastructure, including roads, drainage systems, water, sewerage, and public utilities.

In general, land development is the riskiest but most profitable technique as it is so dependent on the public sector for approvals and infrastructure and because it involves a long investment period with no positive cash flow.

In real estate land value is propably the most fundamental topic. Land is the fundamental characteristic of real estate. The nature of land valuation helps to define the investment characteristics of most real estate assets. The hold of the land has been recognized as an option to develop a completed building at a future date. Option valuation theory (OVT) was developed during the last few decades help to show that the source of vacant land value derives from right, but not the obligation, to develop an underlying asset (a completed building) by paying the relevant exercise price (cost of construction).

3.5 Types of real options in real estate

Opposed to traditional financial options, basically real options underlying assets are real assets. In the case of real estate, typical applications of real option theory is the land development option, which can be seen as an call option. The land development option can give the land owner “the right without obligation to develop (or redevelop) the land upon payment of construction costs” def. by Geltner (2007).

Many types of decisions could be made by using real options theory. The main examples of real options are as follows.

3.5.1 Waiting option

When any key factor in the business is uncertain we can may be able to acquire higher returns by waiting for a certain period of time than we could acquire by acting immediately. In the case of real estate rent may be increasing or decreasing or to choose an optimal development timing of construction.

3.5.2 Growth options (Phasing options)

When the project is phased into more than two steps, the initial investment provides the firm with growth options to be acquired by the second or later investment, given that the first investment turns out to be successful. In other words, by considering the value of growth options, the firm may be able to go ahead with the first project even if that project itself is expected to have a negative return.

3.5.3 Flexibility options (Switching options)

This option refers to the flexibility built into the initial project design. By incorporating flexibility to react to the uncertainty in the future, the project can have higher value than the value based on the traditional DCF analysis. In the case of real estate, what is called “*conversion*” is an example of switching options. In the case of real estate the option to switch the use from hotels to condominiums.

3.5.4 Exit options (Abandonment options)

When there is a certain amount of risk to continue the project in the future, it could be possible to initiate the project, taking into consideration the value of the option to exit from the project when the risk becomes obvious. In the case of real estate, there is an abandonment option for the land owner of vacant land, which is selling the land without a building on it.

3.5.5 Leasing options

When the project can be developed in a phased manner, the firm can test the suitability of the projects by developing the initial phase with low costs. Then, based on the result, the firm can modify (or abandon) the following phase of development in order to maximize the total project value.

3.6 The call option model of land value

It is an applicaton of real options theory to real estate. In this model, land is as obtaining its value through the option it gives its owner (or holder) to develop a structure on the land, Geltner (2007). The landowner can obtain a valuable rent-paying asset upon the payment of the construction cost necessary to built the structure. The model of land value is the most aplicable to vacant land (or nearly vacant land). *The most important is that the real option, that is viewed as giving land its value, is the land development option.* Objective is to evaluate the value of land for a development project. In the other words in OVT it is the right without obligation to obtain land value based on property developed now, upon giving up land value based on future development. By choosing the optimal timing to develop their land, the landowners can maximize its value. An option to develop a certain site now precludes an option to develop the same site later. But normal DCF methods are not able to evaluate this value. The option model can capture the landowners flexibility. There are other options besides developing or not developing (switch, sell) we will focus on the “wait” option.

The fundamental concept of this model is that it can compare the values of the land based on whether it is developed immediately, or if development is postponed. The equation bellow implies that, if the present value (PV) of immediate development is greater than later development, a landowner should commence a project now, in the other words waiting maximizes the option value.

$$OptionValue = Maximum\{\text{PV of the land developed now, PV of the land developed in the future}\}$$

Call option is an option which can be exercised anytime before it expires, in the other words landowners can develop their land anytime they want and this factor, this flexibility, makes calculating the land value based on future development more complex.

3.6.1 A Rigorous model of option value

The call option model of land value demonstrate how a future uncertainty about the market for built property interacts with the irreversibility of the construction process to give the land an option premium and make it optimal, in some cases to delay a project. But we need to know the OCC of the option to wait in order to evaluate the NPV of that alternative and compare it to alternative to building today. The option valuation solve this problem.

3.6.2 The certainly-equivalence perspective on the option value

The DCF method is the traditional risk-adjusted discounting approach, in which risky future cash flow or value amounts are discounted to the present value using a risk-adjusted discount rate that reflects the opportunity cost of capital (OCC) for investments of similar risk to that of the future claim being discounted. This approach is the most widely employed in practice, it is possible to define a different approach that is equivalent but provides a useful additional perspective. This alternative approach is often called *certainly-equivalence valuation* and provides some capabilities that the traditional approach does not. It is optimal to use this approach for large-scale real estate development projects, because it is impossible to know what the correct OCC to apply to the project would be.

By applying the certainty-equivalence formula, we can “risk-adjust” the cash flows which are based on “real” probabilities, and discount the calculated “certainty-equivalent value” at the risk-free rate to adjust it for the time value of money. The equation bellow can show us the link between the risk-adjusted approach and certainly equivalence approach.

$$PV[V_1] = \frac{E_0[V_1]}{1 + E_0[r_v]}$$

$$E_0[r_v] = r_f + RP_v,$$

where r_f risk free rate, RP_v is the market´s required risk premium in the expected total return for the investment, $PV[V_1]$ present value of built property.

We can easily expand this formula, so that the dominator reflects the time value of money, the discounting is done risklessly and the risk is accounted for in the numerator,

$$PV[V_1] = \frac{E_0[V_1]}{1 + E_0[r_v]} = \frac{E_0[V_1]}{1 + r_f + RP_v}$$

$$(1 + r_f + RP_V)PV[V_1] = E_0[V_1]$$

$$(1 + r_f)PV[V_1] + (RP_v)PV[V_1] = E_0[V_1]$$

$$PV[V_1] = \frac{E_0[V_1] - (RP_v)PV[V_1]}{1 + r_f} = \frac{CEQ_0[V_1]}{1 + r_f}, \tag{3.1}$$

where $CEQ_0[V_1]$ certainly equivalent value.

The future value $CEQ_0[V_1]$ is referred to as the certainty equivalent value. Notice that the certainty equivalent value equals the expected value $E_0[V_1]$ less a risk discount $(RP_v)PV[V_1]$. Thats why the certainty equivalent value is the amount such that the investment market would be indefferent between a riskless claim to recieve that amount for certain, and the actual claim to recieve the risky amount V_1 which could turn out to be either greater or less than $E_0[V_1]$. Thus the OCC to use in discounting the $CEQ_0[V_1]$ value is the risk free rate.

To evaluate the option we have to know the present value of the underlying asset (building). If we know the future scenario for the office building, the OCC (opportunity cost of capital) or the current value of the investment asset we can determine the investment present value and expected return question. It is relatively easy to evaluate assets such as the future claim on the office building using risk-adjusted discounting. But it is difficult to observe the OCC or the present value of the option to built the building.

We suppose that the market´s required risk premium in an investment, RP, is the proportional to the risk in the investment as measured by percentage spread in the possible change in value of the investment between now and next year. This percentage spread is the difference between the up outcome (rise in value) and down outcome (fall in value). This outcome spread measures the amount of risk in an investment in the building. Given the building´s required risk premium (equal to tis OCC minus the risk free interest rate), that implies that the market´s risk premium per unit of risk. Also the same price of risk applies to all asstes, and their required return risk premia must all have the same proportion to their risk. We can see this relationship in the equation bellow,

$$PV[V_1]=\frac{E_0[CF_1]}{1+E_0[r]}=\frac{pV_1^{up}+(1+p)V_1^{down}}{1+r_f+RP_V}=\frac{RP_V}{\left(V_1^{up}-V_1^{down}\right)/PV[V_1]}=\frac{RP_V}{\left(V_1^{up}\%-V_1^{down}\%\right)}, \quad (3.2)$$

$$PV[C_1]=\frac{E_0[CF_1]}{1+E_0[r]}=\frac{pC_1^{up}+(1+p)C_1^{down}}{1+r_f+RP_V}=\frac{RP_V}{\left(C_1^{up}-C_1^{down}\right)/PV[V_1]}=\frac{RP_V}{\left(C_1^{up}\%-C_1^{down}\%\right)}, \quad (3.3)$$

where $V_{1,up}$ expected vallue of0 built property when the value increases, $V_{1,down}$ expected vallue of built property when the value decreases, p robability of option value, $C_{1,up}$ expected option value when option value increases, $C_{1,down}$expected option value when option value decreases, $PV[C_1]$ present value of the option, r_v expected annual total return on investment in the underlying asset, RP_c market´s required risk premium of development project, $\left(V_1^{up}\%-V_1^{down}\%\right)$ is the percentage spread, difference between the rise in value and fall in value, measures the amount of risk in an investment, $\left(C_1^{up}\%-C_1^{down}\%\right)$ is the percentage spread, difference between the up and down option movement.

General certainty equivalence formula for the binomial world is as follows,

$$PV[C_1]=\frac{E_0[C_1]}{1+E_0[r_c]}=\frac{E_0[C_1]-\left(C_{1,up}-C_{1,down}\right)\dfrac{E_0[r_v]-r_f}{V_{1,up}\%-V_{1,down}\%}}{1+r_f}. \quad (3.4)$$

Having obtained the present value of the option, we can now back out the opportunity cost of capital of the option $E_0[r_c]$, and the risk premium for the option, RP$_c$,

$$E_0[r_c]=\frac{E_0[C_1]}{PV[C_1]}, \quad (3.5)$$

$$RP_C=E_0[r_c]-r_{f.} \quad (3.6)$$

In equilibrium assets in the investment market must trade at prices that reflects the same price of risk for all assets. Thus the investment expected return risk premium per unit of investment risk musi be the same for the option and for the underlying asset from which it

derivatives its value. In the other words the land value is derivative, based solely on the value of the office building that can be built on it. Also the relative amount of investment risk in an underlying asset and an option that depends upon that asset cen be measured by the ratio of the percentage spreads in the underlying asset´s and the option´s investment returns between the up and down possibilities in outcomes. This is also ussumption for derivatives and their underlying assets, as derivatives must be perfectly correlated with their underlying assets.
We can see the above assumptions in the figure bellow.

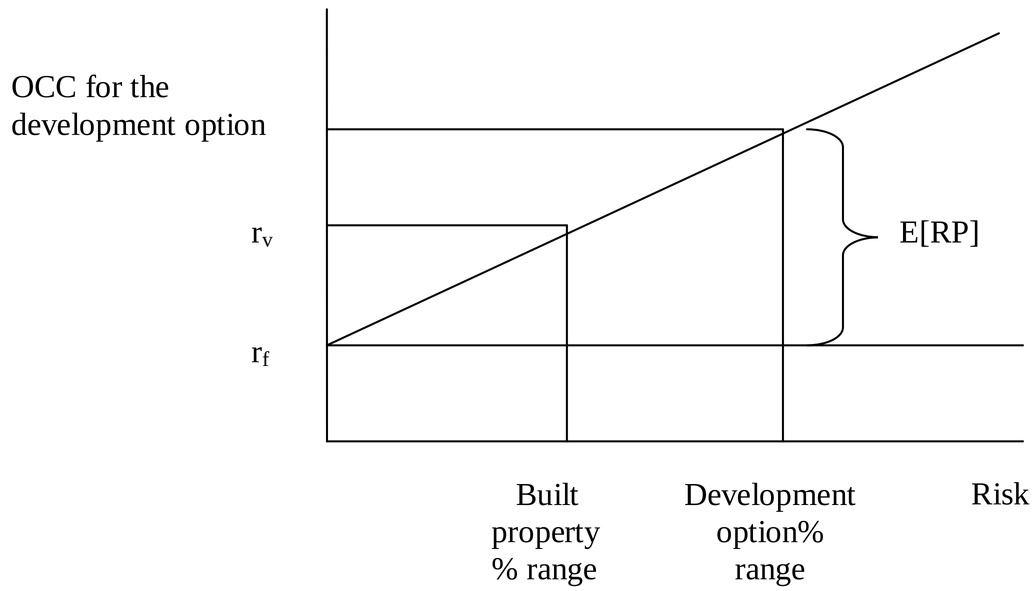


Figure 3.4 Expected return risk premium per unit of risk across the markets for builtproperty and developable land in the option model

3.6.3 Binomial real-option model

This option valuation method evaluates (finite-lived) real options by creating binomical trees, each node of which represents the actual "up" or "down" of values of the underlying asset over time. The charts bellow show the construction of binominal trees.

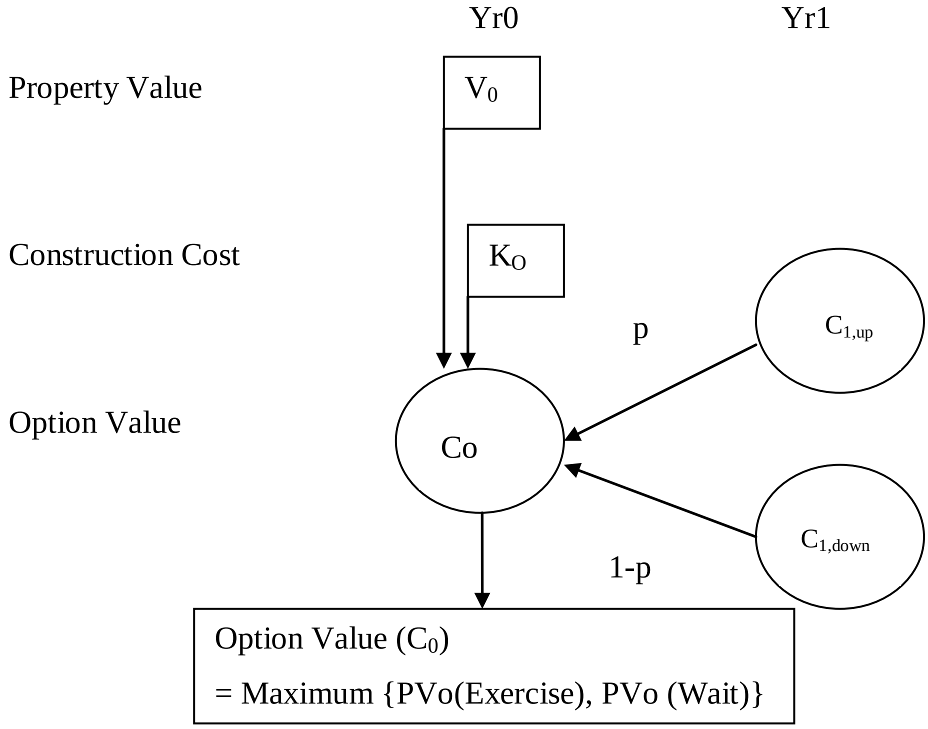


Figure 3.5 One-period binominal model

Option Value (C_o) = Maximum {PV_o (Exercise), PV_o (Wait)}, (3.7)

PV_o (Exercise) = Vo – Ko, (3.8)

$$PV_o \text{ (Wait)} = \frac{ExpectedOptionValue(yr1)}{1+OCC} = \frac{pC_{1,up} + (1-p)C_{1,down}}{1+OCC}, \quad (3.9)$$

where PV_0 *(Exercise)* present value of the land development at time 0, K_0 construction costs.

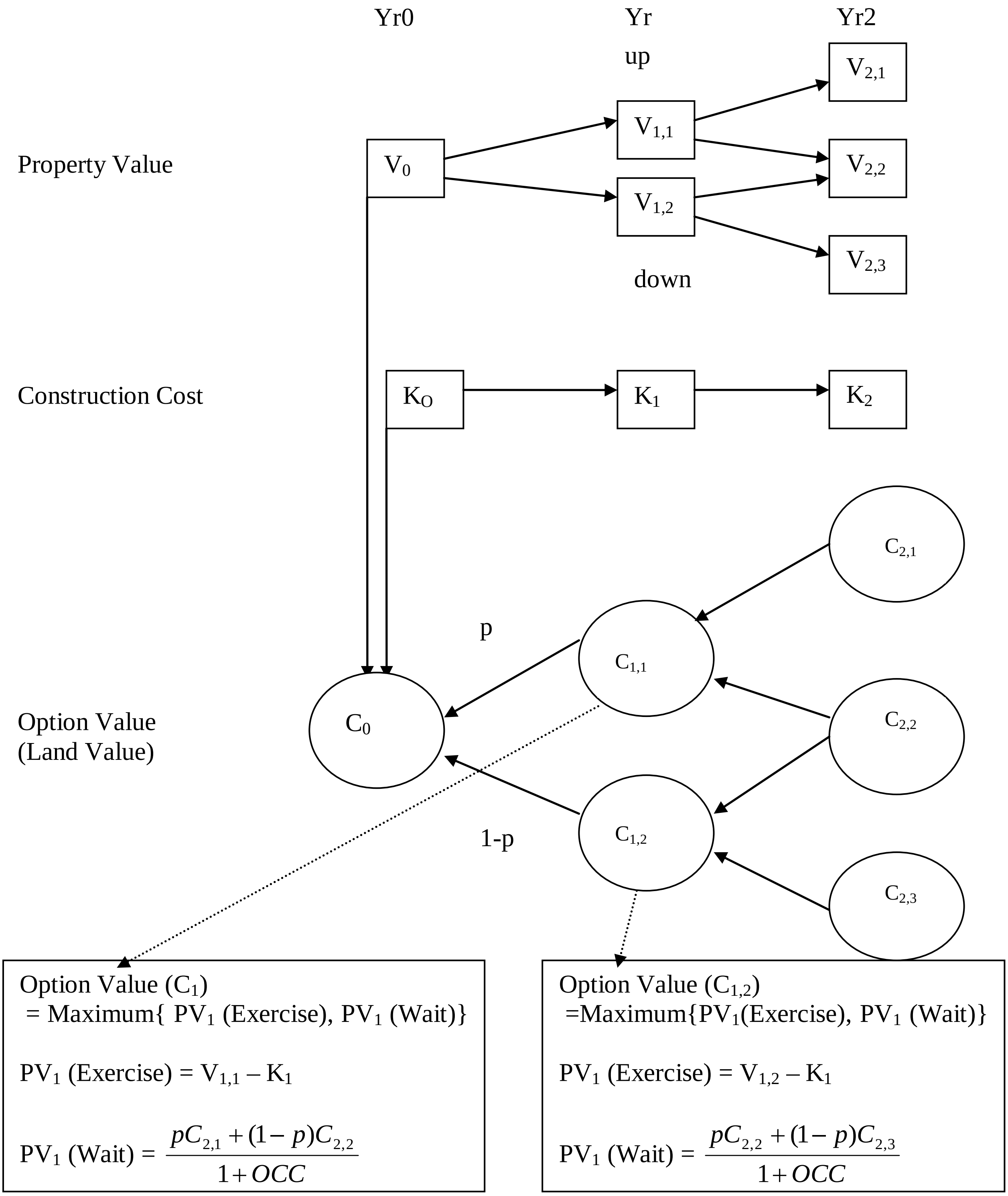


Figure 3.6 Two-period binominal model

The value of the option at each node is expressed by the following formula,

$$\text{Option Value } (\mathrm{C}_{\mathrm{t,j}}) = \text{Maximum } \{\text{PVt (Exercise)}, \text{PVt (Wait)}\}$$

$$= Max\left\{V_{t,i} - K_t ; \frac{pC_{t+1,i} + (1-p)C_{t+1,i+1}}{1+OCC}\right\}$$

$$= Max\left\{ V_{t,i} /(1+y_v)^{ttb} - K_t /(1+y_k)^{ttb} ; \frac{pC_{t+1,i} + (1-p)C_{t+1,i+1} - \dfrac{RPv}{(\%\mathrm{V}_{\mathrm{t,up}} - \%\mathrm{V}_{\mathrm{t,down}})}(C_{t+1,i} - C_{t+1,i+1})}{(1+r_f)} \right\}$$

$$= Max\left\{ V_{t,i} /(1+y_v)^{ttb} - K_t /(1+y_k)^{ttb} ; \frac{pC_{t+1,i} + (1-p)C_{t+1,i+1} - \dfrac{r_v - r_f}{((1+\sigma_v)\text{-}1/(1+\sigma_V))}(C_{t+1,i} - C_{t+1,i+1})}{(1+r_f)} \right\}, \quad (3.10)$$

where σ_v expected annual volatility of returns of the built property, measured by the standard deviation of individual property total returns across time, r_v expected annual total return on investment in the underlying asset,y_k construction cost yield (the diference between the oportunity cost of capital of construction cost cash flows and the expected grow rate in construction costs), y_v annual net rent income cash yield as a fiction of current building value, *ttb* time to built.

For the point at which option expires (for all t=T),

$$OptionValue(\mathrm{Ct, j}) = Max(PV_t(Exercise);0) \quad (3.11)$$

3.6.4 Time to built

To make these methods more realistic, we need to account for the time required between the beginning of construction and the completion of the building. We let “ttb” denote the time required to build the underlying asset.

$$PV_t[V_t] = v_t = \frac{ExpectedVa\,tCompletion}{OCCforBuilt\,\Pr operty} = \frac{V_t(1+g_v)^{ttb}}{(1+r_v)} = \frac{V_t\left(\dfrac{(1+r_v)}{(1+y_v)}\right)^{ttb}}{(1+r_v)^{ttb}} = \frac{V_t}{(1+y_v)^{ttb}} \quad (3.12)$$

$$PV_t[K_t] = k_t = \frac{ExpectedKa\,tCompletion}{OCCforBuilt\Pr operty} = \frac{K_t(1+g_k)^{ttb}}{(1+r_f)} = \frac{K_t\left(\frac{(1+r_f)}{(1+y_k)}\right)^{ttb}}{(1+r_f)^{ttb}} = \frac{K_t}{(1+y_k)^{ttb}}, \quad (3.13)$$

where g_v expected grow rate in built property, g_k expected grow rate in construction costs, *ttb* time to built.

Here we use the "risk free rate" as an opportunity cost for development, because the cash outflow (the negative cash flow) from the development costs has almost no correlation with market portfolio. The payment of all development costs except for land are usually covered by construction loan. This allows us think only at *Time 0* (beginning of the construction) and *Time t* (end of construction) in terms of cash flow.

3.6.5 Present value of exercise option

$$\mathrm{PV_t\,(Exercise)} = \mathrm{PV_t\ (Built\ Property)\text{-}\ PV_t\ (Development\ Costs\,(Excluding\ land))}$$

$$= V_{t,i} - K_t$$

$$= V_{t,i}/(1+y_v)^{ttb} - K_t/(1+y_k)^{ttb} \quad (3.14)$$

$$\mathrm{V_{t+1,up}} = PV_t[V_{t+1}]\cdot \%V_{t+1,up} = PV_t[V_{t+1}](1+\sigma_v) = \frac{V_t(1+g_v)}{(1+r_v)}(1+\sigma_v) = \frac{V_t}{(1+y_v)}(1+\sigma_v), \quad (3.15)$$

$$\mathrm{V_{t+1,down}} = PV_t[V_{t+1}]\cdot \%V_{t+1,down} = PV_t[V_{t+1}]\cdot(1+\sigma_v) = \frac{V_t(1+g_v)}{(1+r_v)}\cdot\frac{1}{(1+\sigma_v)} = \frac{V_t}{(1+y_v)}\cdot\frac{1}{(1+\sigma_v)}, \quad (3.16)$$

where $1+g_v = (1+y_v)/(1+r_v)$.

$$K_{t+1} = K(1+g_k) \quad (3.17)$$

$$PV[Land] = PV[Built\Pr operty] - PV[DevelopmentCost]$$

$$= \frac{V_t - K_t}{(OCCforDevelopment)} = \frac{V_t}{(OCCforBuilt\Pr operty)^{ttb}} - \frac{K_t}{(OCCforDevelopmentCosts)^{ttb}}$$

$$= \frac{V_t - K_t}{(1+r_c)^{ttb}} = \frac{V_t}{(1+r_v)^{ttb}} - \frac{K_t}{(1+r_f)^{ttb}} \quad (3.18)$$

3.6.6 Present value of wait option

$$PV_t(Wait) = \frac{ExpectedOptionValue(1yrLater)}{(1+OCC)} = \frac{pC_{t+1,i} + (1-p)C_{t+1,i+1}}{1+OCC} = \frac{pC_{t+1,i} + (1-p)C_{t+1,i+1}}{1+r_f+RP_c}$$

$$\Downarrow$$

$$(1+r_f+RP)PV_t(Wait) = pC_{t+1,i} + (1-p)C_{t+1,i+1}$$

$$(1+\mathrm{r_f})PV_t(Wait) = (\mathrm{pC_{t+1,i}} + (1-p)C_{t+1,i+1}) - RP_c PV_t(Wait)$$

$$PV_t(Wait) = \frac{(pC_{t+1,i} + (1-p)C_{t+1,i+1}) - RP_c PV_t(Wait)}{(1+r_f)} \ldots\ldots 1$$

$$RPcPV_t(Wait) =$$

$$= \frac{RP_c}{(C_{t+1,up} - C_{t+1,down})}(C_{t+1,up} - C_{t+1,down}) \cdot PV_t(Wait) = \frac{RP_c}{(C_{t+1,up} - C_{t+1,down})/PV_t(Wait)}(C_{t+1,up} - C_{t+1,down})$$

$$= \frac{RP_c}{(\%C_{t+1,up} - \%C_{t+1,down})} = \frac{RP_v}{(\%V_{t+1,up} - \%V_{t+1,down})} \ldots\ldots. 2$$

$$RPcPV_t(Wait) = \frac{RPv}{\%V_{t+1,up} - \%V_{t+1,down})} \cdot C_{t+1,up} - C_{t+1,down}) \ldots\ldots..3$$

By combing 1 and 3 we get

$$PV_t(Wait) = \frac{(C_{t+1,up} - C_{t+1,down}) - RPcPV_t(Wait)}{(1+r_f)}$$

$$= \frac{pC_{t+1,up} + (1-p)C_{t+1,down} - \frac{RPv}{(\%V_{t+1,up} - \%V_{t+1,down})} \cdot \left(C_{t+1,up} + C_{t+1,down}\right)}{(1+r_f)}$$

$$= \frac{pC_{t+1,up} + (1-p)C_{t+1,down} - \frac{r_v - r_f}{((1+\sigma_v) - 1/(1+\sigma_v))} \cdot \left(C_{t+1,up} + C_{t+1,down}\right)}{(1+r_f)} \qquad (3.19)$$

3.6.7 Propability *p*

We use “real” probability approach, which should be distinguished from the risk-neutral probability approach that is more commonly used in economic applications, including in the binominal option valuation model.

The primary advantage of using the risk-neutral probability approach is that we do not need to make an assumption on the risk-adjusted discount rate, and that we can simply use the risk-free rate of interest. However, since it mathematically modifies *up* and *down* probabilities so that cash flows can be discounted at the risk-free rate, it is often difficult for practitioners to understand the method intuitively. Also, since the probabilities are not “true” probabilities related to the actual expected movement of the underlying asset, it is sometimes confusing to illustrate the movement graphically.

We primarily use the “real” probability approach. In the binominal option valuation model, I use the “real” probability approach along with the certainty-equivalence approach. Discounting future cash flows should account for the time value of money and the risk premium. By applying the certainty-equivalence formula, we can “risk-adjust” cash flows which are based on “real” probabilities, and discount the calculated “certainty-equivalent value” at the risk-free rate to adjust it for the time value of money.

The propability can be determined to increase the expected V value one term later to the expected return of built property (r_v).

$$p=\frac{1+r_v-\%V_{1,down}}{\%V_{1,up}-\%V_{1,down}}=\frac{1+r_v-1/(1+\sigma_v)}{(1+\sigma_v)-1/(1+\sigma_v)} \tag{3.20}$$

$$\Uparrow$$

$$1+r_v=p\%V_{1,up}+(1-p)\%V_{1,down}$$

$$p(\%V_{1,up}-\%V_{1,down})=1+r_v-\%V_{1,down}$$

$$\%V_{1,up}=\frac{V_{1,up}}{PV[V_1]}=\frac{V_0(1+\sigma_v)/(1+y_v)}{V_0(1+g_v)/(1+r_v)}=\frac{V_0(1+\sigma_v)/(1+y_v)}{V_0(1+y_v)}=1+\sigma_v$$

$$\%V_{1,down}=\frac{V_{1,down}}{PV[V_1]}=\frac{V_0/(1+\sigma_v)/(1+y_v)}{V_0(1+g_v)/(1+r_v)}=\frac{V_0/(1+\sigma_v)/(1+y_v)}{V_0(1+y_v)}=1/(1+\sigma_v), \tag{3.21}$$

where $1+g_v=(1+y_v)/(1+r_v)$.

3.6.8 Price of risk

One of the most important concept of this model is the “price of risk”, the risk premium per unit of risk. The unit of risk must be the same for the built property and the undeveloped land. In other words, the arbitrage opportunity exists, arbitrage opportunities present “supernormal” profits. Risk, the volatility of value change, can be expressed as the range of expected values.

Price of Risk for Option (Land) = Price of Risk for Built Property

$$\frac{RPc}{(\%C_{up} - \%C_{down})} = \frac{RPv}{(\%V_{up} - \%V_{down})} \qquad (3.22)$$

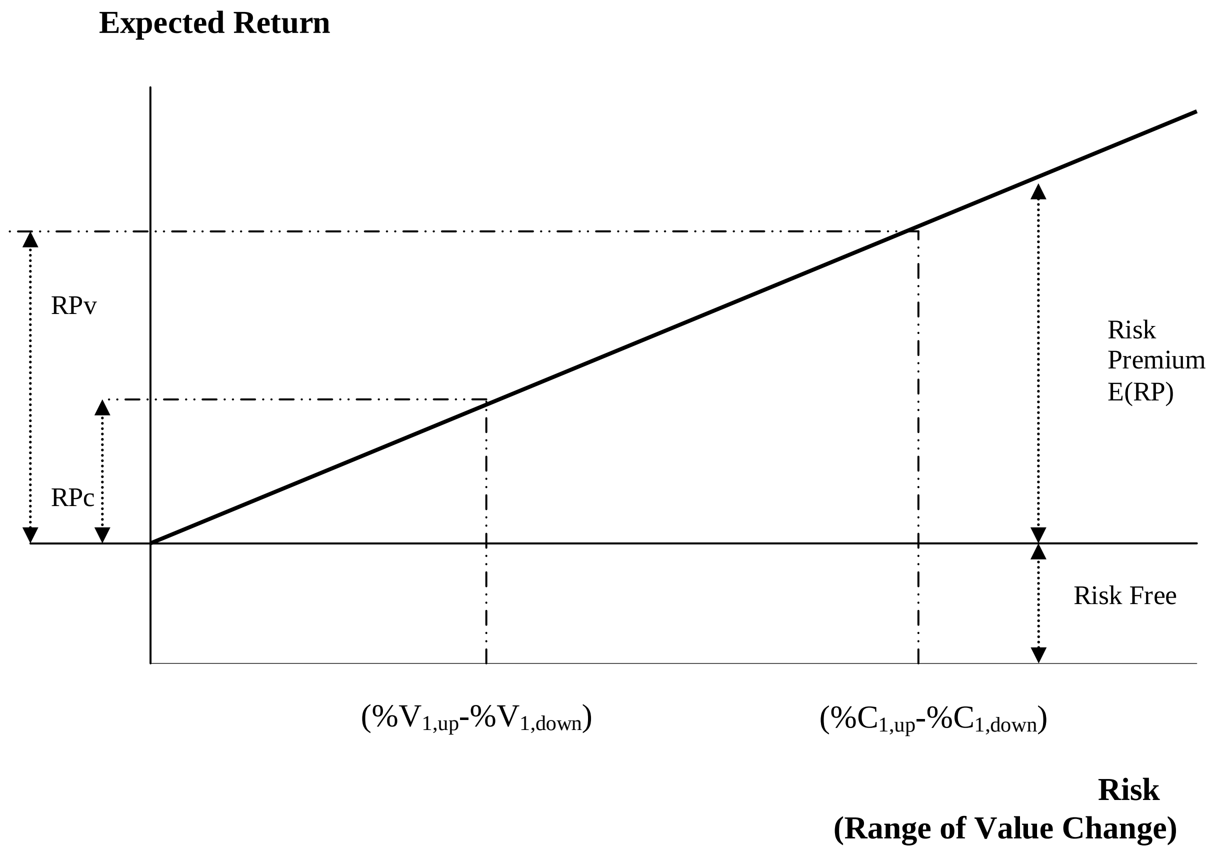


Figure 3.7 Price of Risk

3.7 A perpetual model in continous time

The option valuation model have one important weakness. The binominal trees have to come to an end after some periods. That is to say the land development option should be finite. More often the land development can be seen as an perpetual American call option.

3.7.1 Samuelson-Mc Kean formula

The Samuelson-McKean Formula is an example of the closed-form solutions for the real options, originallly developed for pricing perpetual american warrants (that is a perpetual call option that can be exercised at any time on a dividend-paying underlying asset) by Paul Samuelson and Henry McKean in 1965. Regarding the developable land as a call option without maturity of expiration, the formula is being suitable for valuing real estate development options. This formula is a perpetual model in continous time and can capture values with infinite options, in the other words, land ownership is perpetual and an option to develop a land lasts infinitely.

The formula requires as inputs three parameter values which describe the underlying real estate and construction markets: the built property current cash yield rate, the volatility in the built property value and construction cost yield. The construction cash yield is the difference between the opportunity cost of capital of construction cost cash flows and the expected grow rate in construction costs. Construction yield is expressed by the equation bellow,

$$y_k = r_f - g_k \,, \tag{3.23}$$

where r_f is the risk-free interested rate and g_k is the expected grow rate in the construction costs.

The property´s current cash yield rate (y_v), and the volatility of the built property (σ_v), measured by the standard deviation of individual property total returns across time. The relevant volatility here, for properties already developed and in operation, not vacant land and parcels, is the volatility of individual property. Typical values for this volatility measure would be between 10% ans 25%. Given value all these parametres, we can define,

- *option elasticity* – the measure η is referred to as an option elasticity, because when the option is alived (not yet exercised, the land is not developed yet), the option elasticity gives the percentage change in value of the option (tha land), associated with a 1% change in the value of the underlying asset (built property).

$$\eta = \{y_v - y_k + \sigma_v^2/2 + \left[(y_v - y_k + \sigma_v^2/2)^2 + 2y_k\sigma_v^2\right]^{1/2}\}/\sigma_v^2 \tag{3.24}$$

- *option (land) value* – the other values are the built value (V_0) and construction cost (K_0) of the best proeject that could be built on site.

$$C_0 = (V^* - K_0)\cdot\left(\frac{V_0}{V^*}\right)^{\eta} \tag{3.25}$$

- *hurdle value (critical value)* - suggests the optimal timing of the immediate exercise. It is the value of the developed property bellow which the land should be held undeveloped for the time being and above which it is optimal to develop the land immediately. This hurdle value is a simple function of the current development cost and the option elasticity defined earlier.

$$V^* = K_0\eta/(\eta - 1) \tag{3.26}$$

- *hurdle benefit/cost ratio* – is the ratio of built property value divided by construction cost exclusive of land cost, which triggers immediate optimal development. The ratio is independent of the scale of the project,

$$\eta/(\eta - 1). \tag{3.27}$$

3.7.2 Implication of the model for development timing and landspeculation

The Samuelson-Mc-Kean formula obtain useful insights not only about the value of a given land parcel but also about the optimal timing of tis development. The risk premium in the expected return required by investors holding speculative land should be expressed by following formula,

$$RP_c = \eta PR_v, \tag{3.28}$$

where RP_c is the expected return risk premium in the vacant land holding, RP_v is trhe risk premium for built properties in the underlying real estate asset market.

The relationship among built property volatility and current cash yield, land value and the hurdle benefit/cost ratio, are displayed graphically in the Chart 3.7.

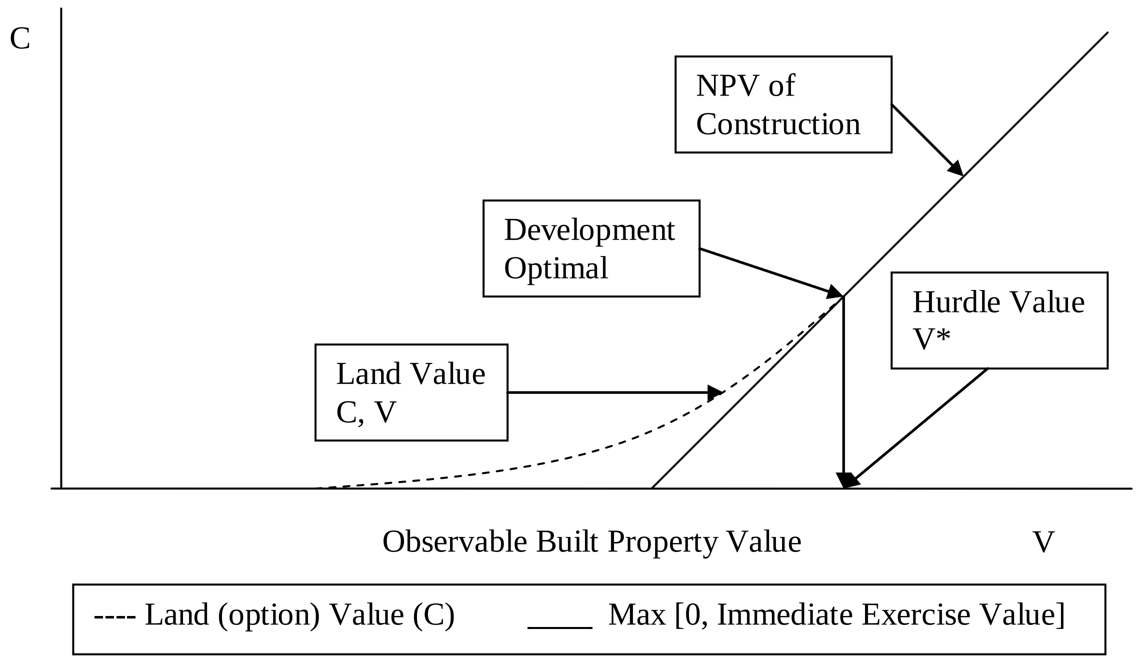


Figure 3.8 Land value as a function of current built property value

3.8 Compound model

It is not realistic to delay a construction, when a developer purchases a site of development, since the decision to develop the site has been made before the purchase.

It is working in the same way as for simple projects with no phases we described before. Benefit of the model is in its aplication to a multi-phased projects. Such projects require a long time to complete, and the development plan must be continually modified regarding the timing of construction depending on the current market situation. These important decisions need to be made not only at the beginning of the project but also periodically. In other words, the later phases have a big deal of flexibility to adapt to market conditions, so far this flexibility has a great impact on a land value.

This model is a method of evaluating land value of multi-phased projects. There are two ways of the model, simultaneous or sequential.

Simultaneous-option model - The phases are independent of each other, and can start anytime.

Sequential-option model - Subsequent phases cannot start until the current phase finishes. We call this vision as an option-on-option model.

The steps to consruct this kind of model are as follows,

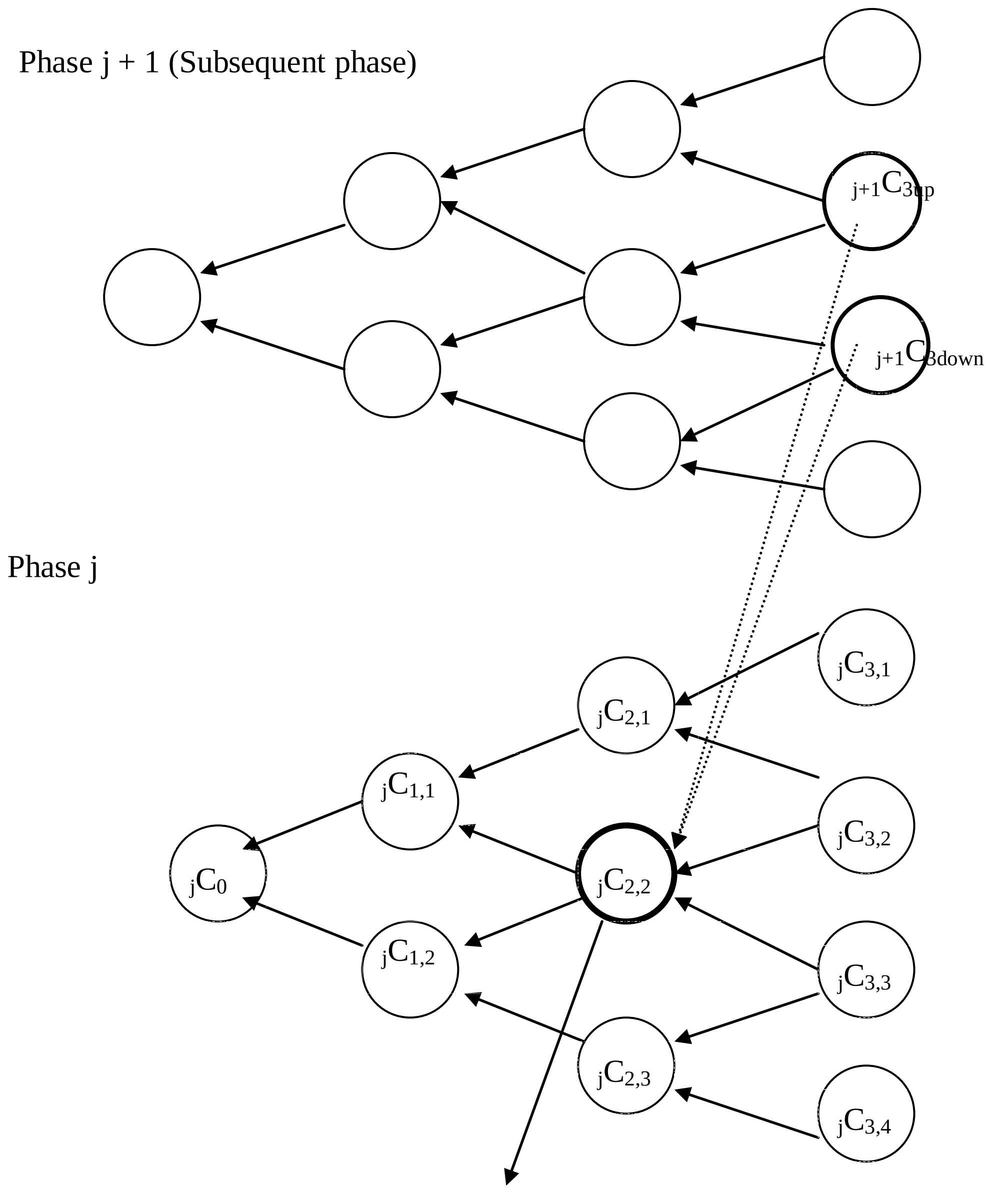
- construct a binomical option model for each phase,
- add an optional value for each subsequent phase only whenn the timing for exercise of the option is optimal. Landowners cannot get the value of subsequent phase untill current phase's comppletion. To incorporate this lag into the evalution, the subsequent option value recieved by current phase can by discounting at the time of completion to the value at the time of exercise.

The following formula and figure is an example of above procedure, assuming one term lag.

OptionValue (${}_jC_{t,i}$) = Max {PV_t(Exercise) + PV_t(Subsequent Phase Option Value), PV_t(Wait)}
PV_t(Subsequent Phase Option Value) =

$$\frac{p_{j+1}C_{t+1,up}+(1-p)_{j+1}C_{t+1,down}}{1+OCC}=\frac{p_{j+1}C_{t+1,up}+(1-p)_{j+1}C_{t+1,down}-\dfrac{RPv}{(\%_{j+1}V_{t+1,up}-\%_{j+1}V_{t+1,down})}\cdot{}_{j+1}C_{t+1,up}+{}_{j+1}C_{t+1,down}}{(1+r_f)}$$

$$=\frac{p_{j+1}C_{t+1,up}+(1-p)_{j+1}C_{t+1,down}-\dfrac{r_v-r_f}{((1+\sigma_v)-1/(1+\sigma_v))}\cdot{}_{j+1}C_{t+1,up}+{}_{j+1}C_{t+1,down}}{(1+r_f)} \tag{3.29}$$



$_jC_{2,2}$=MAX{PV$_2$(Exercise)+PV$_2$(Subsequent Phase Option Value), PV$_2$(Wait)}

PV$_2$(Subsequent Phase Option Value)

$$= \frac{p\,{}_{j+1}C_{3,up} + (1-p)\,{}_{j+1}C_{3,down}}{1+OCC} = \frac{p\,{}_{j+1}C_{3,up} + (1-p)\,{}_{j+1}C_{3,down} - \frac{r_v - r_f}{((1+\sigma_v) - 1/(1+\sigma_v))} \cdot {}_{j+1}C_{3,up} + {}_{j+1}C_{3,down}}{(1+r_f)} \qquad (3.30)$$

Figure 3.9 Compound model

4. Real option aplication in development project

This part of diploma thesis is focused on using the real option valuation method, discussed before, to the real estate development project. We will use a real estate development project which could be realized in the city of Senov.

Senov is small town located in Moravian-Silesia in the Czech republic. It is very strategically situated town between three big cities, with really good public transportation. That's why currently plenty of people are moving into the city and the city is running short of housing. The 20-hectare site is a property of landowner let´s call him Mr. XX. It is situated near the centre of the town and currently zoned to allow 57 units of apartments as a single project, which we call as the Project A and could be realized at anytime. But there is the another project which could be realized, Project B. This alternative idea is based on special exemption, which allows to develop more units than the current city limitation allows, this allowance is limited by time.

Therefore it is important to compute the current value of the site with Project A and the value for the proposed special exemption for the two-phased Project B to get to know how much someone could profitably bid to the landowner for the site, if it is profitable to realize it and when it is optimal for developer to start with the construction.

4.1 The input data

The assumptions of both projects are summarized in table 4.1.

	Project A	Project B	
		Phase 1	Phase 2
of Unit	50	90	150
Built Property Value V_O	45 000 000	66 000 000	110 000 000
Construction Cost K_O	29 000 000	44 000 000	88 000 000
Time to build ttb (months)	12	24	24
Deadline to build (from now)	perpetual	36	60
Construction Cost Grow Rate g_k	2,50%		
Cap Rate y_v	8,00%		
Market OCC for Stabilized Asset r_v	9,00%		
Risk Free Interested Rate r_f	3,50%		
Volatility of Built Property σ_V	20,00%		

*All annual rates are monthly compounding , annual percentage rates.

Table 4.1 The Assumptions of projects

Let‘s take a look at the Project A. For the Moravian-Silesia region current appartment rents in the city of Senov could charge gross rents 8 200CZK/ month, operating expences 10 000/year per occupied unit and average vacancy rate 35,9%[1]. Capital rates (y_v) on such properties are usually 8%[2]. So we can compute the property value if it existed today:

$$\Pr opertyValu e(\mathrm{Re}nts \cdot 12 - OperatingExpences) \cdot (1 - VacancyRat e) / CapRate \cdot Units \qquad (4.1)$$
$$= (8200 \cdot 12 - 10000) \cdot 0{,}6410 / 0{,}08 \cdot 57 = 45000000CZK$$

By using the equation (4.1) we compute that the property would be worth 45 000 000CZK. Construction costs would be 32 000 000CZK with a projected construction grow rate 2,5%. Construction would take one year.

We can see also assumptions for both parts of the Project B defined by following conditions,

- Phase 1 can be developed at any time between now and 36 months from now.
- Phase 2 can be developed at anytime within 5 years from now, but only after Phase 1 has been developed,
- allowance of much greater density on the site, with rents under the market value.

4.2 Rael option approach

Here we review the real option value for both projects based on the real-option approach, in the other words we use the binomial option valuation method.

On the Figure 4.1 we can see the structure of the land-development problem.

[1] http://www.stavebni-forum.cz/diskuse2009/prezentace/0611_sumera.pdf
[2] http://www.netgainrealestate.com/cap-rate-recommendations

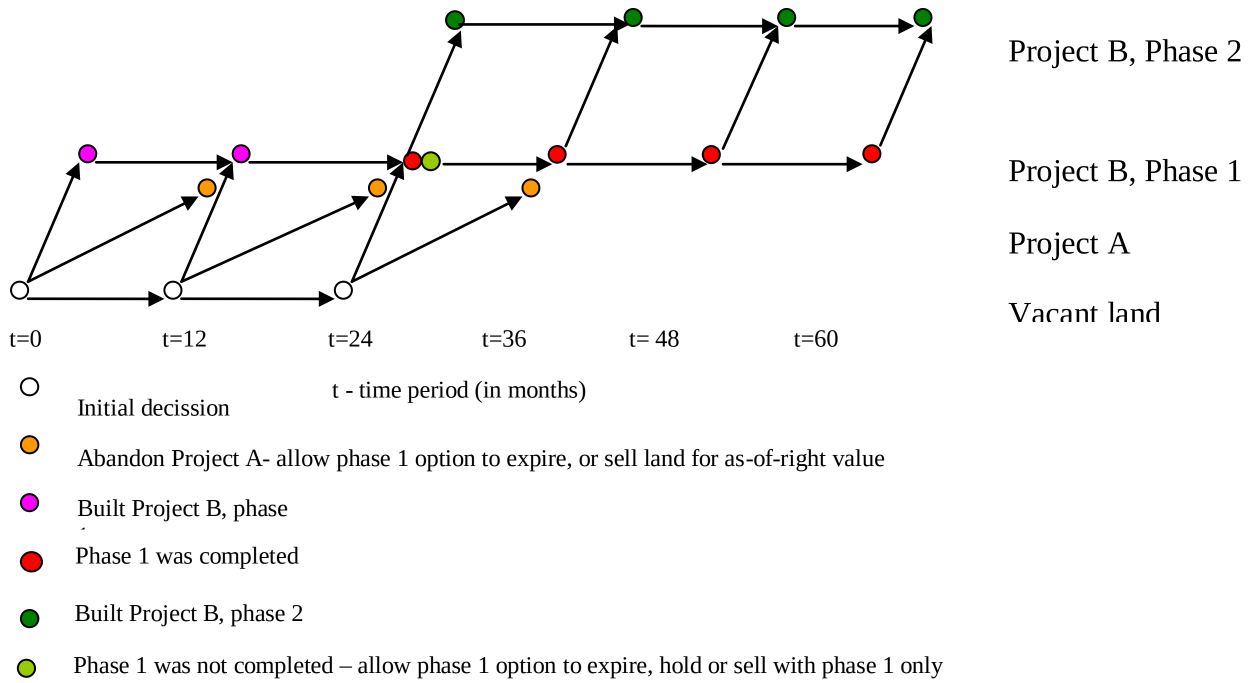


Figure 4.1 Decssion tree for the land-development problem

We need to know the current value of the site for the Project A, because this is the opportunity cost for the Project B, necessary to compute the NPV of this project. Futures values represents the abandonment values of the Project B, in the other words the values of the site if the developer decides not to purchase the project.

For the computation we use the Samuelson-McKean formula for perpetual option, because the project could be developed anytime.

We need to know the value of this 2-phased Project B, because it suggests how much someone could profitably bid for the project. And it allows a computation of the additional value created by the special exemption to provide more units on the site, necessary to compute the NPV of the project.

We will see that the model allow us to find when it is optimal for developer to start with the construction and also find the opportunity cost of capital for investment.

To compute the option value of 2-phased project we use the coumpound model. Phase 1 can be developed at any time between now and 36 months from now, this means that developing Phase 1 is an American call option. Phase 2 of the project can be exercised at anytime within 60 months from now, but only after Phase 1 has been developed, it is also an American call option, therefore the Project B is characterized as a compound option, where the underlying asset of the Phase 1 option includes the option of Phase 2.

If the developer finds the Project B unprofitable, he can abandon the right of special exemption and sell the land. The price of land would be based on as-of-right value of the Project A development value. We can call this alternative as an “abandonment option”. If Phase 2 is not developed after Phase 1 is completed, the land has the value of Phase 1, since value of Phase 1 exceeds the value of the Project A.

The possible results could be:

- develop Phase 1 and Phase 2 of the Project B,
- develop only Phase 1 of the Project B,
- abandon the Project B and obtain the as-of-right land value based on the Project A value.

Now we can start to compute the values of the land with development projects.

Project A

First we calculate the value of abandonment option, in the other words the as-of-right value. As we discussed before we use the Project A witch could be started at any time (also represents the opportunity costs of the other project). Therefore we can use the Samuelson Mc-Kean formula, the equation 3.21 to compute option elasticity,

$$\eta = \frac{0{,}6666 - 0{,}0831 + 5{,}7735^2/2 + \sqrt{0{,}0831 - 0{,}6666 - 5{,}7735^2/2}}{5{,}7735^2} = 4{,}6 .$$

Now we have to develop a binomial trees in the way I have discussed in chapter 3.

In other words at the first we develop the tree of underlying asset values. If we observe the current value of underlying asset V_0, the values of underlying asset at one year from now are calculated as follows,

$$V_{1,1} = V_0(1+\sigma_v)/(1+y_v) \quad \text{(up case)}, \tag{4.3}$$

$$V_{1,2} = V_0/(1+\sigma_v)/(1+y_v) \quad \text{(down case)}. \tag{4.4}$$

The propability is due to equation (3.17),

$$p = ((1+0{,}0075) - 1/(1+0{,}05577))/((1+0{,}0577) - 1/(1+0{,}0577)) = 0{,}55.$$

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):											
0	45 000 000	47 282 857	49 681 524	52 201 876	54 850 085	57 632 639	60 556 352	63 628 385	66 856 263	70 247 892	73 811 579
1		42 261 990	44 405 947	46 658 668	49 025 670	51 512 750	54 126 000	56 871 820	59 756 937	62 788 415	65 973 682
2			39 690 573	41 704 082	43 819 736	46 042 719	48 378 473	50 832 721	53 411 473	56 121 045	58 968 075
3				37 275 613	39 166 610	41 153 539	43 241 264	45 434 901	47 739 820	50 161 669	52 706 378
4					35 007 590	36 783 531	38 649 565	40 610 264	42 670 429	44 835 107	47 109 599
5						32 877 565	34 545 449	36 297 945	38 139 345	40 074 161	42 107 129
6							30 877 140	32 443 542	34 089 408	35 818 769	37 635 861
7								28 998 430	30 469 525	32 015 249	33 639 388
8									27 234 030	28 615 617	30 067 292
9										25 576 985	26 874 509
10											24 020 761

Table 4.2 Underlying asset value tree (only the first 10 months are shown to conserve the space)

The tree for the construction costs is made in similar way. There is no need to use the propability of up and down movements, because we have not volatility in construction costs. We simply increase the construction cost at each step in the tree by the expected grow rate,

$$K_{j+1} = K_j \times (1 + g_k). \tag{4.2}$$

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):											
0	29 000 000	29 060 417	29 120 959	29 181 628	29 242 423	29 303 345	29 364 393	29 425 569	29 486 872	29 548 303	29 609 862
1		29 060 417	29 120 959	29 181 628	29 242 423	29 303 345	29 364 393	29 425 569	29 486 872	29 548 303	29 609 862
2			29 120 959	29 181 628	29 242 423	29 303 345	29 364 393	29 425 569	29 486 872	29 548 303	29 609 862
3				29 181 628	29 242 423	29 303 345	29 364 393	29 425 569	29 486 872	29 548 303	29 609 862
4					29 242 423	29 303 345	29 364 393	29 425 569	29 486 872	29 548 303	29 609 862
5						29 303 345	29 364 393	29 425 569	29 486 872	29 548 303	29 609 862
6							29 364 393	29 425 569	29 486 872	29 548 303	29 609 862
7								29 425 569	29 486 872	29 548 303	29 609 862
8									29 486 872	29 548 303	29 609 862
9										29 548 303	29 609 862
10											29 609 862

Table 4.4 Construction cost tree (only the first 10 months are shown to conserve the space)

Next we calculate the tree of hurdle value using equation 3.23,

$$V_{i,j}* = K_j \cdot \frac{(1+g_k)}{(1+r_f)} \cdot \eta / (\eta - 1) = \frac{K_j}{(1+r_v)} \cdot 1{,}28.$$

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):											
0	36 667 232	36 743 622	36 820 171	36 896 880	36 973 748	37 050 777	37 127 966	37 205 316	37 282 827	37 360 500	37 438 334
1		36 743 622	36 820 171	36 896 880	36 973 748	37 050 777	37 127 966	37 205 316	37 282 827	37 360 500	37 438 334
2			36 820 171	36 896 880	36 973 748	37 050 777	37 127 966	37 205 316	37 282 827	37 360 500	37 438 334
3				36 896 880	36 973 748	37 050 777	37 127 966	37 205 316	37 282 827	37 360 500	37 438 334
4					36 973 748	37 050 777	37 127 966	37 205 316	37 282 827	37 360 500	37 438 334
5						37 050 777	37 127 966	37 205 316	37 282 827	37 360 500	37 438 334
6							37 127 966	37 205 316	37 282 827	37 360 500	37 438 334
7								37 205 316	37 282 827	37 360 500	37 438 334
8									37 282 827	37 360 500	37 438 334
9										37 360 500	37 438 334
10											37 438 334

Table 4.5 Hurdle value tree (only the first 10 months are shown to conserve the space)

Then we calculate the options value starting from the terminal period (month 60). We suppose that we do not develop the land and wait untill month 60, then our decision is,

- start construction at month 60,
- or abandon the project.

Therefore the option values at the terminal moment T are calculated as maximum of immediate exercise or abandonment project,

$$C_{i,T} = MAX\left(V_{i,T} - K_T ;0\right) . \tag{4.3}$$

Then for the periods before the option´s expiration the option values should be equal to the maximum of either:

- start construction at each period,
- or wait until next period.

To compute waiting values we use certainly-equivalence formula. Then the option values are calculated as follows,

$$C_{i,t} = MAX\left(V_{t,i}/(1+y_v)^{ttb} - K_t/(1+y_k)^{ttb} , \frac{pC_{t+1,i} + (1-p)C_{t+1,i+1} - \frac{r_v - r_f}{((1+\sigma_V)-1/(1+\sigma_V))} \cdot (C_{t+1,i} - C_{t+1,i+1})}{(1+r_f)} \right) . \tag{4.4}$$

Considering one year time-to-built the land value in each node is as follows.

If $\dfrac{V}{(1+y_v)^{12}} \leq V^*$,

Land $= V^* - K/\left(1+y_k\right)^{12}\Big) \cdot \left(\dfrac{V/\left(1+y_v\right)^{12}}{V^*}\right)^{\eta}$.

If $\frac{V}{(1+y_v)^{12}} > V^*$,

$$\text{Land}=\frac{V}{\left(1+y_v\right)^{12}}-\frac{K}{\left(1+y_k\right)^{12}}. \tag{4.5}$$

The binomial tree of the land price based on the as-of-right Project A. This tree is used as an abandonment value tree in analysis of Project B.

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):											
0	12 839 104	14 887 190	17 042 085	19 309 214	21 694 276	24 203 262	26 842 463	29 618 492	32 538 295	35 609 173	38 838 796
1		10 251 114	12 170 820	14 190 829	16 316 236	18 552 392	20 904 924	23 379 740	25 983 051	28 721 380	31 601 584
2			7 818 260	9 615 955	11 509 278	13 501 577	15 597 880	17 803 469	20 123 894	22 564 988	25 132 877
3				5 810 142	7 244 088	8 987 096	10 854 379	12 819 330	14 886 909	17 062 329	19 351 068
4					4 317 809	5 383 446	6 712 083	8 364 446	10 206 028	12 143 987	14 183 217
5						3 208 781	4 000 709	4 988 086	6 219 149	7 754 037	9 564 130
6							2 384 606	2 973 128	3 706 898	4 621 762	5 762 415
7								1 772 121	2 209 481	2 754 782	3 434 664
8									1 316 952	1 641 977	2 047 217
9										978 693	1 220 235
10											727 316

Table 4.6 Land value tree (only the first 12 months are shown to conserve the space)

Project B

Next we start to develop binomical trees of Project B. Considering that Phase 1 of Project B it is a compound option including the option value of Phase 2. We discussed compound option model in the chapter 3. We first built the binomical trees of Phase 2. Built the underlying asset value tree, corresponding construction cost tree and option value tree. We have to work backward to built option value tree using the certainly equivalence formula. We have to discount the values one year back, because the option of Phase 2 can only be exercised after twelve months of the option exercise of Phase 1.

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
Expected Values:		110 091 060	110 182 195	110 273 405	110 364 691	110 456 052	110 547 490	110 639 002	110 730 591	110 822 255	110 913 996
0	110 000 000	115 580 318	121 443 725	127 604 585	134 077 986	140 879 783	148 026 637	155 536 052	163 426 421	171 717 069	180 428 303
1		103 307 086	108 547 871	114 054 521	119 840 525	125 920 054	132 307 999	139 020 005	146 072 512	153 482 793	161 269 000
2			97 021 400	101 943 311	107 114 911	112 548 868	118 258 490	124 257 762	130 561 378	137 184 777	144 144 183
3				91 118 164	95 740 603	100 597 539	105 700 868	111 063 090	116 697 339	122 617 413	128 837 814
4					85 574 109	89 915 297	94 476 715	99 269 534	104 305 493	109 596 927	115 156 797
5						80 367 380	84 444 430	88 728 310	93 229 511	97 959 059	102 928 538
6							75 477 453	79 306 437	83 329 665	87 556 992	91 998 772
7								70 885 052	74 481 062	78 259 498	82 229 615
8									66 572 074	69 949 286	73 497 825
9										62 521 518	65 693 245
10											58 717 417

Table 4.7 Underlying asset value tree (only the first 10 months are shown to canserve the space)

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):											
0	88 000 000	88 183 333	88 367 049	88 551 147	88 735 628	88 920 494	89 105 745	89 291 382	89 477 406	89 663 817	89 850 617
1		88 183 333	88 367 049	88 551 147	88 735 628	88 920 494	89 105 745	89 291 382	89 477 406	89 663 817	89 850 617
2			88 367 049	88 551 147	88 735 628	88 920 494	89 105 745	89 291 382	89 477 406	89 663 817	89 850 617
3				88 551 147	88 735 628	88 920 494	89 105 745	89 291 382	89 477 406	89 663 817	89 850 617
4					88 735 628	88 920 494	89 105 745	89 291 382	89 477 406	89 663 817	89 850 617
5						88 920 494	89 105 745	89 291 382	89 477 406	89 663 817	89 850 617
6							89 105 745	89 291 382	89 477 406	89 663 817	89 850 617
7								89 291 382	89 477 406	89 663 817	89 850 617
8									89 477 406	89 663 817	89 850 617
9										89 663 817	89 850 617
10											89 850 617

Table 4.8 Construction cost tree (only the first 10 months are shown to conserve the space)

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):											
0	12 839 104	14 887 190	17 042 085	19 309 214	21 694 276	24 203 262	26 842 463	29 618 492	32 538 295	35 609 173	38 838 796
1		10 251 114	12 170 820	14 190 829	16 316 236	18 552 392	20 904 924	23 379 740	25 983 051	28 721 380	31 601 584
2			7 818 260	9 615 955	11 509 278	13 501 577	15 597 880	17 803 469	20 123 894	22 564 988	25 132 877
3				5 810 142	7 244 088	8 987 096	10 854 379	12 819 330	14 886 909	17 062 329	19 351 068
4					4 317 809	5 383 446	6 712 083	8 364 446	10 206 028	12 143 987	14 183 217
5						3 208 781	4 000 709	4 988 086	6 219 149	7 754 037	9 564 130
6							2 384 606	2 973 128	3 706 898	4 621 762	5 762 415
7								1 772 121	2 209 481	2 754 782	3 434 664
8									1 316 952	1 641 977	2 047 217
9										978 693	1 220 235
10											727 316

Table 4.9 Present value of 24 months delayed receipt of Phase 2 option value (only the first 10 months are shown to conserve the space)

Finally we can built binomical trees of the Phase 1. In the same way as before. That is to built the underlying asset value tree, corresponding construction cost tree and option value tree. At the option expiration period (month 36), the option value is the maximum of,

- the as-of-right land value (Project A land value),
- immediate exercise of Phase 1 option value, including the present value of Phase 2 compound option.

$$C_{36} = Max\{As - of - rightLandValue_{36}; PV_{36}[V_{60} - K_{60}] + PV_{36}[Phase^{second}Opt_{60}]\} \quad (4.6)$$

The value in any earlier month is the current value of:

- the as-of-right land value (Project A land value),
- immediate exercise of Phase 1 option value, including the present value of Phase 2 compound option,
- holding the option unexercised for at least one more month.

$$C_t = Max\{As - of - rightLandValue_{36}; PV_t[V_{t+24} - K_{t+24}] + PV_t[Phase^{second}Opt_{t+24}]; PV_t[C_{t+12}]\} (4.7)$$

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
Expected Values:		66 054 636	66 109 317	66 164 043	66 218 815	66 273 631	66 328 494	66 383 401	66 438 355	66 493 353	66 548 397
0	66 000 000	69 348 191	72 866 235	76 562 751	80 446 792	84 527 870	88 815 982	93 321 631	98 055 852	103 030 241	108 256 982
1		61 984 251	65 128 722	68 432 713	71 904 315	75 552 033	79 384 799	83 412 003	87 643 507	92 089 676	96 761 400
2			58 212 840	61 165 987	64 268 947	67 529 321	70 955 094	74 554 657	78 336 827	82 310 866	86 486 510
3				54 670 899	57 444 362	60 358 524	63 420 521	66 637 854	70 018 403	73 570 448	77 302 688
4					51 344 466	53 949 178	56 686 029	59 561 720	62 583 296	65 758 156	69 094 078
5						48 220 428	50 666 658	53 236 986	55 937 707	58 775 435	61 757 123
6							45 286 472	47 583 862	49 997 799	52 534 195	55 199 263
7								42 531 031	44 688 637	46 955 699	49 337 769
8									39 943 245	41 969 572	44 098 695
9										37 512 911	39 415 947
10											35 230 450

Table 4.10 Underlying asset value tree (only the first 10 months are shown to conserve the space)

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("i"):											
0	44 000 000	44 091 667	44 183 524	44 275 573	44 367 814	44 460 247	44 552 873	44 645 691	44 738 703	44 831 909	44 925 308
1		44 091 667	44 183 524	44 275 573	44 367 814	44 460 247	44 552 873	44 645 691	44 738 703	44 831 909	44 925 308
2			44 183 524	44 275 573	44 367 814	44 460 247	44 552 873	44 645 691	44 738 703	44 831 909	44 925 308
3				44 275 573	44 367 814	44 460 247	44 552 873	44 645 691	44 738 703	44 831 909	44 925 308
4					44 367 814	44 460 247	44 552 873	44 645 691	44 738 703	44 831 909	44 925 308
5						44 460 247	44 552 873	44 645 691	44 738 703	44 831 909	44 925 308
6							44 552 873	44 645 691	44 738 703	44 831 909	44 925 308
7								44 645 691	44 738 703	44 831 909	44 925 308
8									44 738 703	44 831 909	44 925 308
9										44 831 909	44 925 308
10											44 925 308

Table 4.11 Construction cost tree (only the first 10 months are shown to conserve the space)

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):											
0	27 043 384	31 858 485	36 925 029	42 255 789	47 864 182	53 764 308	59 970 982	66 499 769	73 367 027	80 589 940	88 186 566
1		20 943 941	25 456 790	30 205 764	35 202 857	40 460 672	45 992 450	51 812 105	57 934 254	64 374 260	71 148 263
2			15 220 228	19 435 305	23 886 012	28 569 721	33 498 269	38 684 092	44 140 256	49 880 490	55 919 221
3				10 889 888	13 995 292	17 941 445	22 330 820	26 950 116	31 811 013	36 925 783	42 307 319
4					7 697 633	9 939 713	12 806 436	16 462 146	20 790 986	25 346 709	30 140 837
5						5 391 863	6 992 138	9 048 405	11 678 804	15 027 129	19 266 288
6							3 745 518	4 877 055	6 343 390	8 236 457	10 670 376
7								2 581 005	3 368 126	4 395 615	5 732 682
8									1 770 798	2 310 496	3 019 400
9										1 215 279	1 580 716
10											839 225

Table 4.12 Present value of 24 months delayed receipt of Phase 1 option value (only the first 10 months are shown to conserve the space)

The present value of the land with Project B is 27 043 384 CZK, in other words the gross value of Project B option (fair bid). The value is equal to the immediate exercise value of Phase 1 (13 140 468 CZK) plus the present value of Phase 2 option (12 839 104 CZK), includes consideration of the Project A (abandonment value) and the value of the option to wait and invest later in Phase 1.

The result shows that it is optimal to start the construction of Poject B Phase 1 now (27 043 384>12 839 104). The incremental value added by developing more units, the economic NPV is 14 204 280 CZK (27 043 384 – 12 839 104).

The model can also shows us when it is optimal for developer to start the construction.

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):											
0	exer	exer	exer	exer	exer	exer	exer	exer	exer	exer	exer
1		exer	exer	exer	exer	exer	exer	exer	exer	exer	exer
2			exer	exer	exer	exer	exer	exer	exer	exer	exer
3				exer	exer	exer	exer	exer	exer	exer	exer
4					exer	exer	exer	exer	exer	exer	exer
5						exer	exer	exer	exer	exer	exer
6							exer	exer	exer	exer	exer
7								exer	exer	exer	exer
8									exer	exer	exer
9										exer	exer
10											exer

Table 4.13 Phase 1 option optimal decission tree (only the first 10 months are shown to conserve the space)

The option model also allows us to quantify the opportunity cost of capital for investment in the multi-phased project using the following procedure,

$$C_{i,j} = \frac{CEQ\left[C_{j+1}\right]}{1+r_f} = \frac{E_j\left[C_{j+1}\right]}{1+r_f+E\left[RP_{Ci,j}\right]} = \frac{E_j\left[C_{j+1}\right]}{1+OCC_{i,j}} \Rightarrow 1+OCC_{i,j} = \left(1+r_f\right)\frac{E_j\left[C_{j+1}\right]}{CEQ\left[C_{j+1}\right]} \; .$$

Thus, for the Project B, the current OCC is given by,

$$1+OCC_{0,0} = \left(1+r_f\right)\frac{E_0\left[C_1\right]}{CEQ_0\left[C_1\right]} \; . \tag{4.9}$$

Year ("j"):	0	1	2	3	4	5	6	7	8	9	10
"down" moves ("*i*"):											
0	1,98%	1,79%	1,65%	1,53%	1,44%	1,37%	1,30%	1,25%	1,20%	1,16%	1,13%
1		2,34%	2,06%	1,85%	1,70%	1,57%	1,47%	1,39%	1,33%	1,27%	1,22%
2			2,58%	2,42%	2,15%	1,92%	1,75%	1,62%	1,51%	1,42%	1,35%
3				2,65%	2,62%	2,50%	2,26%	2,00%	1,81%	1,66%	1,55%
4					2,70%	2,68%	2,65%	2,57%	2,37%	2,08%	1,87%
5						2,75%	2,73%	2,70%	2,66%	2,63%	2,45%
6							2,79%	2,78%	2,76%	2,74%	2,71%
7								2,82%	2,82%	2,81%	2,80%
8									2,82%	2,83%	2,84%
9										2,78%	2,82%
10											2,71%

Table 4.14 Project B tree of OCC (only the first 10 months are shown to conserve the space)

The OCC for the Project B is currently 1,98% per month.

As discussed in the chapter (3) this enables us to rigorously quantify the amont of investment risk in the Project B, relative to that in the relevant built property assets.

$$\frac{E(RPc)}{E(RPv)} = \frac{OCC-r_f}{r_v-r_f} = \frac{1{,}98\% - 0{,}29167\%}{0{,}75000\% - 0{,}29167\%} = 3{,}67\% \tag{4.10}$$

Thus, the Project B currently has over 3 times the amount of investment risk (as evaluated by the capital market) as does an investment in a completed apartment property.

4.3 DCF approach

Next, I conduct conventional DCF analysis. To see the difference between the real option approach.

- each phase will begin as soon as possible,
- 1,00% discounted rate is assumed (usually a rate near 1% per period has often been applied),
- the net cash flows would be obtained in month 24 and 48 from the expected values.

Expected values of the built property $= V_0\left((1+r_v)/(1+y_v)\right)^t$ (4.11)

To compute the present value we use the equation bellow and discounting the cash flow we get a gross PV for the Project B.

$$\left(\frac{E\left(V_{24}^{phase1}\right)}{(1+r_v)^{24}}-\frac{K_{24}^{phase1}}{\left(1+r_f\right)^{24}}\right)+\left(\frac{E\left(V_{50}^{phase2}\right)}{(1+r_v)^{50}}-\frac{K_{50}^{phase2}}{\left(1+r_f\right)^{50}}\right)=16\,594\,251+10\,675\,956=27\,270\,207\text{CZK}$$

Time(year)	0	12	24	36	48
Phase 1 Built Property Value			67 323 817		
Phase 1 Construction Cost			46 253 522		
Phase 2 Built Property Value					114 456 980
Phase 2 Construction Cost					97 244 925
Net Cash Flow			21 070 295		17 212 055
Discount Rate	1%				
PV	27 270 207				

Table 4.16 Present Value based on DCF analysis

Thus, the DCF analysis would suggest a gross present value of 27 270 207 CZK for the site with special exemption.

4.4 Comparison of real option and DCF based approaches

Using both approaches we assumed different bid prices of the site with special exemption to develop more units. Table 4.9 shows us the results.

	Bid Price (PV of the project)	OCC	Land cost	NPV
Real Option Approach	27 043 384	1,98%	12 839 104	14 204 280
Traditional DCF Analysis	27 270 207	1,00%		14 431 102

Table 4.17 Real option & DCF approach

Using real option model to evaluate the multi-phased project we get the value of the option 27 043 384 CZK. Thus, our model of option value is telling us that it is in fact optimal for the developer to immediately begin construction of Phase 1 of the Project B with special zoning exemption. We see here one of the useful features of the option model of project valuation, that it not only tells us the value of the project, but also whether it is optimal to start with the construction.

In summary the real options value theory has included the opportunity cost of capital, the value of flexibility and phasing in the possible development projects, in a model based on the concept of market equlibrium across the market for built property, land and bonds. The analysis allows us to say a fair price for taking of the site with the Project A would be 12 839 104 CZK. A fair bid for the site with the proposed special exemption for the two-phased Project B would be 27 043 384 CZK. Net present value of the project would be 14 204 280 CZK. Developer of the site with the special exemption would immediatelly begin the construction on Phase 1. The option model allows us to quantify the opportunity cost of capital, thus 1,98% per annum, and allows us to say that the development project has over 3 times the amount of investment risk as does an unlevered investment in a completed apartment property.

The conventional DCF analysis suggests a bid price of 27 270 207 CZK. Compare to the real option analysis, DCF approach overestimate the project value. NPV of the project would be 14 431 102 CZK, due to the basic NPV rule developers should start with the construction.

5. Conclusion

The real estate land value it is propably the most fundamental topic. Land is the fundamental characteristic of real estate. The nature of land valuation helps to define the investment characteristics of most real estate assets. The hold of the land has been recognized as an option to develop a completed building at a future date. Option valuation theory (OVT) was developed during the last few decades helps to show that the source of vacant land value derived from the right, but not the obligation, to develop an underlying asset (a completed building) by paying the relevant exercise price (cost of construction).

The main aim of the diploma thesis was to evaluate the value of land for the development project using the real option theory.

The first chapter was a simple introduction and also adumbration of the diploma thesis.

The second chapter was firstly devoted to a general characteristic of financial and real options. It means theirs most important features, types of option contract and payoff diagrams. The second part of this chapter was focused on real option methodology. More deeply were described the approaches determining the value of the option.

The following chapter included a description of real estate system and development-decision making process. The second part of this chapter was focused on land value, the fundamental characteristic of real estate. It was explained how to evaluate the land by using the call option model (binomial real-option model, perpetual model in continous time and compound model).

The fourth chapter can be named as a practical part. We separately computed the values of the site with development projects. We needed to know the current value of the site with the Project A because this was the opportunity cost for the site with Project B (with special zoning exemption). For the computation we used the Samuelson-McKean formula for perpetual option. After we have used the compound model to get to know a fair bid for the site with the proposed special exemption for the two-phased Project B, the value was 27 043 384 CZK. Finally we used traditional DCF analysis to get to know a fair bid for the site with the proposed special exemption for the two-phased Project B, to see the difference between these two valuation approaches, the value was 27 270 207 CZK.

The fifth chapter gave us briethly conclusion.

Summarizing the key considerations between the real options approach and DCF valuation, we can say, that the option approach is more rigorous, based fundamentally on a market equilibrium, provides a more-correct valuation, provides a deeper understanding of the sources of the project value and true nature of the project investment risk and return. In the other words, the model tells us, what the value of the land is, what its price should be, in order to provide a fair expected return to investment. On the other side, the DCF analysis is based on ad hoc assumptions regarding the project execution and opportunity cost of capital and ignoring its flexibility. At the end we can say that if the conventional opproach gives the same result as the options approach, there is no way to know that the valuation is correct or why it is correct if it is correct.

References

Books, working papers

ARNOLD, T., & CRACK, T. (2003), “Option Pricing in the Real World: A Generalized Binominal Model with Applications to Real Options,” Department of Finance, University of Richmond, Working Paper

GELTNER, C., MILLER, N., *Commercial Real Estate Analysis & Investment.* 2nd edition. South-Western College Publishing Co., 2007. 848p. ISBN 0-324-30548-6.

BRUEGGEMAN, W. B., FISHER, J. D., *Real Estate Finance and Investments,* 13th edition. International Edition, 2008. 688 p. ISBN 978-007-125919-4.

DLUHOŠOVÁ, D. a kol., *Nové přístupy a finanční nástroje ve finančním rozhodování*. Ekonomická fakulta VŠB-TUO, 2004. 640 s. ISBN 80-248-0669-X.

HULL, John. *Options, futures and other derivatives.* 3rd ed. New Jersey: Prentice Hall International Editions, 1997. 572 p. ISBN 0-13-264367-7.

SCHWARTZ, E. S., TRIGEORGIS, L. Real Options *and Investment under Uncertainty*, Cambridge, Massachusetts, London: MIT Press, 2001. ISBN 0-262-19446-5.

TISON, E., GRISWOLD, R. S., *Real Estate Investing for Dummies,* 2nd edition. Willey Publishing, Inc., 2009. 384p. ISBN 978-0-470-28966-2.

ZMEŠKAL, Zdeněk. *Financial models.*1st ed. Ostrava: VSB-TUO Ostrava, 2004. 254 p. ISBN 80-248-0754-8.

b) internet webpages

www.stavebni-forum.cz
www.netgainrealestate.com

List of Abbreviations

A_t	sale price
C_t	option value (land value) at time t
$C_{t,down}$	option value when option value decreases at time t
$C_{t,up}$	option value when option value increases at time t
DCF	discounnted cash flow
e.g.	exempli gratia
g_v	grow rate in built property
g_k	grow rate in construction costs
IV	intrictic value
I	investment cost
K_t	construction costs and other costs to develop a property at time t,
Max	maximum
NPV	net present value
OCC	oportunity cost of capital
p	propability
p_d	propability (moving down)
p_u	propability (moving up)
R	revenue
r_f	risk free rate
r_v	expected annual total return on investment in the underlying asset
S	underlying asset
t	time
ttb	time to built
VC	variable cost
V_t	value of built property at time t
$V_{t,down}$	e xpected vallue of built property when the value decreases at time t
$V_{t,up}$	expected value of built property when the value increases at time t
X	strike price
y_k	construction cost yield
y_v	annual net rent income cash yield as a fiction of current building value
WACC	weighted average cost of capital

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V Ostravě dne 30. 4. 2010

……………………………….
Lenka Musialová

Adresa trvalého pobytu studenta:

Krátká 1263
Šenov, 739 34

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